Exam 9

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CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam 9

Financial Risk and Rate of Return

May 3, 2018

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 60.5-point examination consists of 21 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid/tape.

   - Write your Candidate ID number and the examination number, 9, at the top of each answer sheet. For your Candidate ID number, four boxes are provided corresponding to one box for each digit in your Candidate ID number. If your Candidate ID number is fewer than 4 digits, begin in the first box and do not include leading zeroes. Your name, or any other identifying mark, must not appear.
   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
   - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam, you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that the table of the Normal Distribution is attached to the examination after the last question.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 17, 2018.

END OF INSTRUCTIONS

1. (11 points)

A portfolio manager is looking to make recommendations to his clients using the Capital Asset Pricing Model (CAPM). The research department has developed the following information:

	Forecast Return (%)	Standard Deviation (%)	Beta
Stock X	15.0	25	0.8
Stock Y	18.0	20	1.2
Stock Z	12.5	30	0.9
Market index	14.0	15	
Risk-free rate	5.0		

- Investor preference is given by the following utility formula: $U = E(r) - 1/2A\sigma^2$.
- Investor preference for Stock Z is -0.049 for a risk-averse investor.
- The correlation between Stocks X and Y is 0.6.
- The CAPM expected return for Stock X is 12.2%.
- The implied alpha for Stock Y is 2.2%.
- The Sharpe ratios for Stock X and Stock Z are 0.40 and 0.25, respectively.

a. (4.25 points)

Using CAPM, compare and contrast which stock the portfolio manager would recommend to an investor holding a single stock portfolio versus an investor considering adding one of these stocks to an already well-diversified portfolio. For the single stock portfolio recommended, demonstrate that the risk-reward tradeoff is a better alternative than investing in the market index.

b. (2.25 points)

For both a risk-averse investor and a risk-neutral investor, justify which stock each would select.

c. (2 points)

Graph the capital market line, the efficient frontier, the market index, and Stocks X, Y, and Z. Label the axes and all data points. Briefly explain why Stock X cannot lie on the efficient frontier. Explain how the portfolio manager would interpret the perceived value of the price for Stock Y.

d. (2.5 points)

Demonstrate that the insurance principle of diversification is achieved for the minimum-variance portfolio comprised only of Stocks X and Y.

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2. (7.25 points)

Given the following information regarding an insurance company and the market:

- Loss and Loss Adjustment Expense (LAE) is $75 million.
- Loss and LAE are paid at the following times:

Time (years)	% of Loss and LAE Paid
1	50%
2	30%
3	20%

- 100% of assets are invested at the investment income yield of 5.5%.
- Investment income is paid annually at the end of the year.
- The beta value for this company is 1.5.
- Expenses (other than LAE) are $39 million.
- Premium is collected and expenses (other than LAE) are paid at time 0.
- Premium is immediately earned, so there is no unearned premium reserve.
- Loss reserves are established at time 0.
- An undiscounted reserves-to-surplus ratio of 2:1 is maintained at all times.
- A Treasury Bill is selling for the following price:

Treasury Bill	Price
Face value $1,000, 2-yr maturity, 4% annual coupon	$1,048.90

- The expected market return is 8.5%.
- The risk-free rate is constant over time.
- Ignore taxes and inflation.

There are two profit requirements:

- Regulators require the present value of underwriting cash flows to be no greater than 7% of premium.
- Shareholders require a minimum internal rate of return consistent with CAPM.

Assuming the same business is written every year (steady-state), the Product Manager has requested the business to be priced to achieve a total steady-state calendar year return on surplus of 30%.

Recommend a premium the company should charge. Describe how the recommended premium relates to the premiums needed to satisfy the regulators, the shareholders, and the Product Manager.

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3. (2.75 points)

Given the following for a portfolio:

Stock	Beta	Capitalization ($)
A	1.10	20,000
B	1.25	50,000
C	0.90	30,000

- The risk-free rate is 5%.
- The market price of risk is 1.5.
- The standard deviation of the market portfolio is 20%.

a. (1.5 points)

Using the traditional Capital Asset Pricing Model (CAPM), calculate the expected rate of return on the portfolio.

b. (0.75 point)

Explain the zero-beta version of the CAPM.

c. (0.5 point)

Assume the zero-beta version of the CAPM holds. The expected rate of return on the zero-beta portfolio is 8%. Calculate the expected rate of return on the above portfolio.

CONTINUED ON NEXT PAGE

4. (3.75 points)

A company with the following obligations is concerned about interest rate risk.

Obligation	Amount	Years until due
1	18,708	1.5
2	10,226	3.5

Only the following securities are available:

- Security A: 2-year bond with a 6% coupon rate, paid annually, selling at par.
- Security B: 4-year zero-coupon bond.

The company is looking to immunize their obligations once over the remaining lifetime of their obligations using securities A and B.

a. (2 points)

Calculate amounts invested in the immunizing portfolio.

b. (1.25 points)

Calculate the convexity of the immunizing portfolio.

c. (0.5 point)

A second portfolio becomes available with the following characteristics:

- Duration equal to duration of obligations
- Convexity = 6.74

Recommend and justify which portfolio the company should invest in.

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5. (1.25 points)

   a. (0.5 point)

      Briefly describe the difference between the semi-strong form and strong-form efficient market hypotheses.

   b. (0.75 point)

      Describe a scenario with abnormal returns that would support the semi-strong form but not the strong-form efficient market hypothesis.

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6. (4 points)

Assume the following:

- Portfolios A, B, and C are well-diversified.
- The market is influenced by just two factors, X and Y.

Portfolio	Expected Return	β_X	β_Y
A	15.0%	1.0	0.50
B	7.5%	-0.1	0.25
C	13.5%	0.3	0.75

a. (1.25 points)

Calculate the risk-free rate and risk premiums for factors X and Y.

b. (2.75 points)

Suppose there exists another well-diversified portfolio D with:

- $\beta_X = 0.2$
- $\beta_Y = 0.5$
- Expected return = 12%

Prove that an arbitrage opportunity exists, and construct a strategy to exploit the opportunity that returns $10,000.

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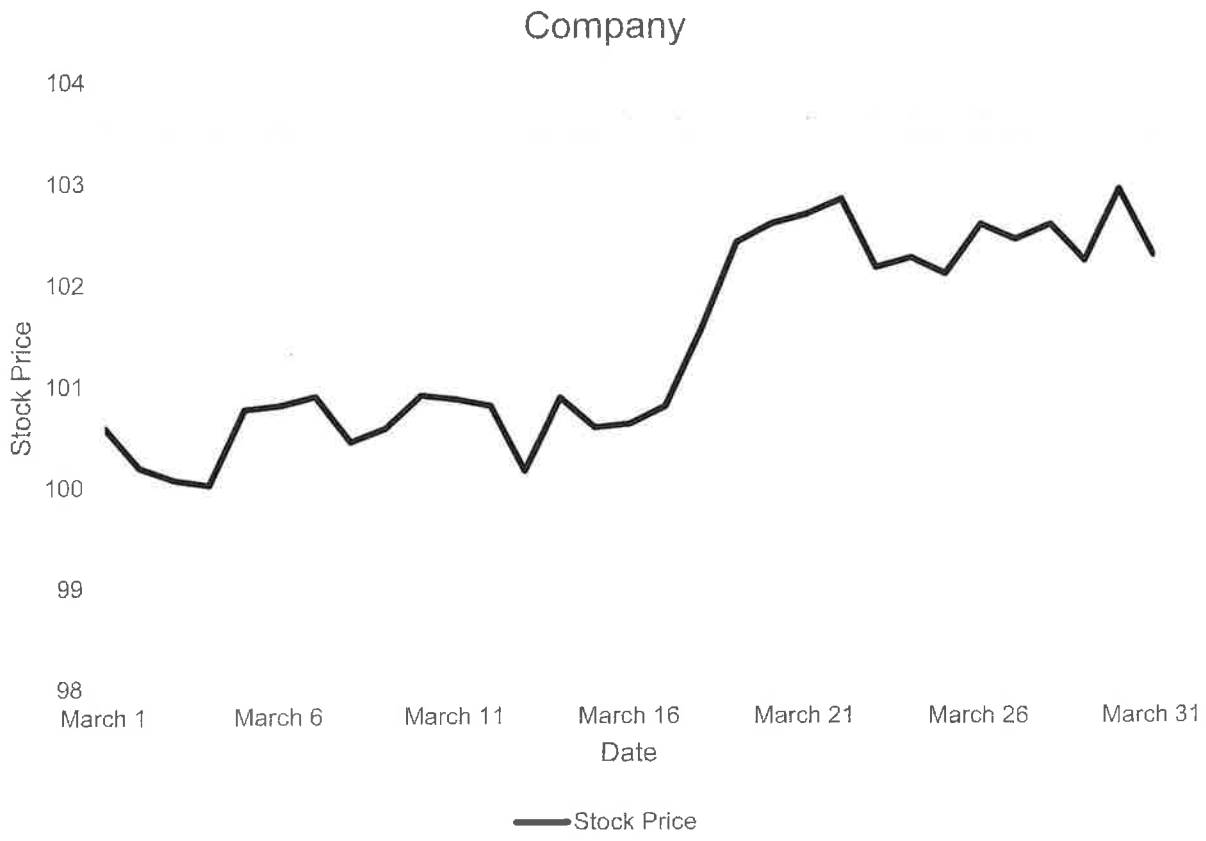
7. (1.5 points)

On March 16 the financial results of a company were released showing stronger performance than the market expected. The chart below shows the stock price during the month of March.



a. (0.5 point)

Identify and briefly describe an information processing error shown above.

b. (1 point)

Identify and briefly describe two limitations that could have prevented a rational investor from profiting from the information processing error that existed after the announcement.

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8. (2 points)

Given the following pure yield curve of zero-coupon Treasuries:

Maturity (Years)	Yield to Maturity
1	7%
2	8%

Consider a newly issued two-year bond that has face value of $1,000 and pays coupons annually of $90 at the end of the year.

a. (0.5 point)

Calculate the issue price of the two-year bond using the pure yield curve given above.

b. (0.25 point)

Calculate the forward rate for the second year.

c. (0.5 point)

Assume the expectations hypothesis is valid. Calculate the expected price of the two-year bond at the beginning of the second year.

d. (0.75 point)

Assume the liquidity preference theory is valid and the liquidity premium is 3%. Calculate the expected price of the two-year bond at the beginning of the second year.

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9. (1.75 points)

A company is evaluating an active bond management strategy to generate abnormal returns. Given the following:

Bond	X	Y	Z
Bond Type	US Treasury	US Treasury	Corporate
Bond Maturity (Years)	15	5	5
Bond Rating	AAA	AAA	AAA
Current Yield	4%	4%	6%
Long Term Historical Yield	5%	5%	6%

- The yield curve is flat.
- The bonds listed above were issued today.
- The bonds listed above are the only investment instruments available.
- All bonds are zero-coupon bonds.
- The company holds $1,000 in par value for each of X and Y.
- The company expects treasury rates to rise from 4% to 6% in 5 years.

a. (0.5 point)

Describe the transactions associated with a rate anticipation swap.

b. (0.75 point)

Calculate the additional assets that the company will own at the end of year 5 under the rate anticipation swap compared to the current portfolio.

c. (0.5 point)

Describe another active strategy the bond manager can use to generate abnormal returns based on a perceived mispricing of bonds.

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10. (1.5 points)

An investor is deciding between investing in the bonds issued by two companies operating in the same industry using the information listed below:

	Company A	Company B
Revenue	$20 million	$50 million
Interest Expense	$2 million	$5 million
Earnings before interest, taxes	$4 million	$7 million
Assets	$10 million	$25 million

a. (1 point)

Based on the companies' financials listed above, use two financial ratios to identify the company that is less likely to default.

b. (0.5 point)

Briefly describe two protective covenants that can be added in the bond contract that will reduce loss to the investor when either company A or B default.

CONTINUED ON NEXT PAGE

11. (2 points)

A specialty property insurer with a geographic footprint that has low correlation to the market's geographic footprint is considering issuing an insurance-linked security to reduce its exposure to hurricane risk.

a. (1.5 points)

Identify two types of CAT bond triggers and briefly describe which one may be preferable from the perspective of the issuer and which one may be preferable from the perspective of the investor.

b. (0.5 point)

Discuss two reasons why an Industry Loss Warranty may be preferable to a non-indemnity CAT bond for this insurer.

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12. (2 points)

An investor is considering investing in the senior tranche of a Collateralized Debt Obligation Squared (CDO^2) composed of the junior tranches of two Collateralized Debt Obligations (CDOs).

Given the following:

- CDOs are composed of two 1-year securities, which pay $100 at maturity and $0 at default.
- Each CDO and CDO^2 is constructed with a junior and senior tranche of width $100.
- The probability of either security defaulting is 15%.
- The discount rate is 5%.

a. (0.75 point)

Calculate the price of the senior tranche of the CDO^2 assuming defaults of the underlying securities are uncorrelated.

b. (0.5 point)

Calculate the price of the senior tranche of the CDO^2 assuming defaults of the underlying securities are perfectly correlated.

c. (0.75 point)

A rating agency uses default probabilities to assign ratings to these securities. Describe whether the expected return of these securities should be higher or lower than the expected return of single-name securities with the same rating.

CONTINUED ON NEXT PAGE

13. (1.25 points)

An insurer has the following risk elements and correlations:

Risk Element	Amount (in millions)	Capital Ratio
Stocks	$ 200	0.20
Bonds	$ 1,000	0.05
Total Assets	$ 1,200	

Risk Element	Amount (in millions)	Capital Ratio
Loss Reserves	$ 800	0.35
Unearned Premium Reserves	$ 100	0.20
Total Liabilities	$ 900	

Correlated Risk Elements	Correlation Coefficient
Stocks - Bonds	0.2
Bonds - Loss Reserves	0.4

Assume no other correlations exist.

a. (0.75 point)

Calculate the diversified risk-based capital held by the insurer.

b. (0.5 point)

Briefly describe an advantage and a disadvantage of the Expected Policyholder Deficit in allocating capital.

CONTINUED ON NEXT PAGE

14. (2.5 points)

An insurer is planning to write a line of business. It plans on releasing capital as the losses are paid at the end of each year, according to the following schedule:

Year	% Paid
1	20%
2	50%
3	30%

Given the following:

- Premiums are $25 million.
- Expense ratio is 15%.
- Expected Losses are $21 million.
- Initial Capital is $6.5 million.
- Investment yield is 4%.
- Single-year target RAROC is 16%.

a. (1 point)

Calculate the multi-year adjusted target RAROC.

b. (1 point)

Using a risk-adjusted return approach, determine if the insurer should write the line of business.

c. (0.5 point)

Briefly describe two approaches for handling different exposure horizons in risk capital models.

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15. (2 points)

An insurer has exposure to two independent perils, wind and earthquake:

- Wind has a 15% chance of a $5 million loss, and an 85% chance of no loss.
- Earthquake has a 1% chance of a $15 million loss, and a 99% chance of no loss.

Using the capital allocation by percentile layer methodology with a 99.5% VaR capital requirement, determine how much capital should be allocated to each peril.

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16. (4.5 points)

The co-measures table below displays simulated values of the aggregate loss distribution for a company in descending order. The component loss sources underlying the aggregate loss are displayed for each scenario.

Sorted Scenarios	Aggregate Underwriting Loss	Line A Underwriting Loss	Line B Underwriting Loss
100	1,000	700	300
99	500	400	100
98	200	0	200
97	100	0	100
96	0	0	0
95	0	0	0
94	0	0	0
93	0	0	0
92	0	0	0
91	0	0	0
⋮	⋮	⋮	⋮
1	0	0	0

- Ignore other risks for the company.
- The target return on capital is 15%.

a. (3 points)

Calculate the 98% VaR risk capital allocated to Line A in proportion to the 98% VaR, the 98% Co-TVAR, and the allocation by Percentile Layer method.

b. (0.75 point)

Explain the characteristics of each methodology above that gives rise to the different results.

c. (0.75 point)

Calculate premium net of expenses for Line A using the 98% VaR risk capital allocated using the Percentile Layer method.

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17. (3 points)

Single Writing Company (SWC) is considering writing a policy with the following characteristics:

Premium (in $000's)	100
Loss (in $000's)	72
Expense (in $000's)	30
Investment Return Rate	6%
Tax Rate	35%
Surplus as % of Present Value of Unpaid Loss	30%
PV Loss Discount for Surplus Calculation	8%
PVI/PVE Discount Rate	10%

Premium, losses, expenses, and investable assets are given:

Year	Earned Premium	Incurred Losses	Incurred Expenses	Paid Losses	Investable Assets
0	0	0	18	0	120
1	100	72	12	36	75
2	0	0	0	36	0
Total	100	72	30	72	195

a. (2.75 points)

Calculate PVI/PVE for this policy.

b. (0.25 point)

Briefly explain under what circumstances PVI/PVE will equal IRR.

CONTINUED ON NEXT PAGE

18. (1.25 points)

Given the following table:

Company	Proposed Premium	Expected Loss and Expense	Premium-to-Surplus Ratio
A	105	94	2:1
B	105	94	1:1
C	98	94	3:1

Each company writes the same coverage, in the same market, and provides the same level of coverage.

a. (0.75 point)

The regulator has determined that a 15% return on surplus is the appropriate benchmark for excessive profitability. Identify any companies whose proposed premium will be rejected by the regulator.

b. (0.5 point)

Describe two disadvantages of the return-on-equity method compared to the return-on-sales method.

CONTINUED ON NEXT PAGE

19. (2.25 points)

An insurance company writes two lines of business. The company has recently decided to purchase reinsurance on Line B while continuing to assume all the risk on Line A. Given the following:

- Company management has a target surplus of 1.2 times the average negative income in the cases where it is below the 1% level.
- Company surplus = 8,000,000.
- Mean investment income on surplus = 350,000.
- Mean underwriting result for line A = 475,000.
- Mean underwriting result for line B = 375,000.
- Line B net cost of reinsurance = 125,000.

%	Income is Below	Mean Value of TVAR				
		Total	Line A	Line B	Investment	Reinsurance
0.1	(6,200,238)	(6,278,992)	(2,172,504)	(4,990,202)	(762,087)	1,645,800
0.2	(5,555,683)	(5,794,174)	(2,017,695)	(4,550,118)	(727,019)	1,500,658
0.4	(4,897,611)	(5,329,696)	(1,871,014)	(4,058,038)	(739,011)	1,338,367
1.0	(4,008,814)	(4,562,868)	(1,652,339)	(3,435,305)	(608,209)	1,132,986
2.0	(3,300,000)	(3,996,893)	(1,450,891)	(2,954,906)	(565,643)	974,547
5.0	(2,307,688)	(3,142,100)	(1,133,566)	(2,305,593)	(463,340)	760,399
10.0	(1,494,462)	(2,361,762)	(857,697)	(1,804,880)	(294,446)	595,261

a. (1.75 points)

Determine the mean return on allocated surplus net of reinsurance for Line B.

b. (0.5 point)

After purchasing reinsurance on Line B the company re-evaluates its surplus position. Determine the amount of total surplus the company could release.

CONTINUED ON NEXT PAGE

20. (1.75 points)

A company writes in two distribution channels, A and B. At the start of a new year, the company decides to allocate a risk load to each renewing distribution channel using the covariance share method and the table of events below. The calculated risk load for channel A is $2.024 million. The calculated risk load for channel B is $1.070 million.

Event	Probability	Channel A Loss (million)	Channel B Loss (million)	Covariance Share A	Covariance Share B
1	0.005	24	8	1.43	0.48
2	0.010	20	10	2.64	1.32
3	0.010	15	15	2.23	2.23
4	0.015	15	7	2.12	0.99
5	0.020	12	7	2.08	1.21
6	0.025	8	9	1.65	1.86
Total				12.15	8.09
Variance		16.76	7.19		

a. (0.5 point)

Determine the risk load multiplier used to calculate the risk loads.

b. (1.25 points)

The company purchases a reinsurance contract that caps losses in distribution channel A at $15 million. Assuming the company maintains the same risk load multiplier, calculate the change in the overall risk load.

CONTINUED ON NEXT PAGE

21. (1.25 points)

Given the following formula: $\frac{T}{S} = \frac{I}{A} * \left(1 + \frac{R}{S}\right) + \frac{U}{P} * \frac{P}{S}$

a. (0.75 point)

Identify the three returns that make up the equation and, for each return, briefly explain whose viewpoint is represented.

b. (0.5 point)

In the equation it could appear that premium (P) has no effect on the total return on surplus (T/S). Briefly describe two possible interactions between the components of this equation that would make this conclusion incorrect.

END OF EXAMINATION

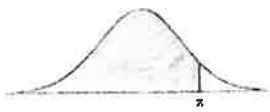
Exam 9
Financial Risk and Rate of Return

May 3, 2018

POINT VALUE OF QUESTIONS

QUESTION	VALUE OF QUESTON	SUB-PART OF QUESTION (a)	(b)	(c)	(d)	(e)	(f)	(g)
1	11.00	4.25	2.25	2.00	2.50			
2	7.25	7.25						
3	2.75	1.50	0.75	0.50				
4	3.75	2.00	1.25	0.50				
5	1.25	0.50	0.75					
6	4.00	1.25	2.75					
7	1.50	0.50	1.00					
8	2.00	0.50	0.25	0.50	0.75			
9	1.75	0.50	0.75	0.50				
10	1.50	1.00	0.50					
11	2.00	1.50	0.50					
12	2.00	0.75	0.50	0.75				
13	1.25	0.75	0.50					
14	2.50	1.00	1.00	0.50				
15	2.00	2.00						
16	4.50	3.00	0.75	0.75				
17	3.00	2.75	0.25					
18	1.25	0.75	0.50					
19	2.25	1.75	0.50					
20	1.75	0.50	1.25					
21	1.25	0.75	0.50					
22	0.00							
23	0.00							
24	0.00							
25	0.00							
26	0.00							
27	0.00							
28	0.00							
29	0.00							
30	0.00							
31	0.00							
32	0.00							
33	0.00							
34	0.00							
35	0.00							
36	0.00							
37	0.00							
38	0.00							
39	0.00							
40	0.00							
41	0.00							
42	0.00							
43	0.00							
44	0.00							
45	0.00							
TOTAL	60.50							

Tables of the Normal Distribution



Probability Content from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Values of z for selected values of Pr(Z<z)

z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

GENERAL COMMENTS:

- Candidates should note that the instructions to the exam explicitly say to show all work; graders expect to see enough support on the candidate's answer sheet to follow the calculations performed. While the graders made every attempt to follow calculations that were not well-documented, lack of documentation may result in the deduction of points where the calculations cannot be followed or are not sufficiently supported.
- Candidates should justify all selections when prompted to do so. For example, if the candidate selects an all year average and the question prompts a justification of all selections, a brief explanation should be provided for the reasoning behind this selection. Candidates should note that a restatement of a numerical selection in words is not a justification.
- Incorrect responses in one part of a question did not preclude candidates from receiving credit for correct work on subsequent parts of the question that depended upon that response.
- Candidates should try to be cognizant of the way an exam question is worded. They must look for key words such as "briefly" or "fully" within the problem. We refer candidates to the Future Fellows article from December 2009 entitled "The Importance of Adverbs" for additional information on this topic.
- Some candidates provided lengthy responses to a "briefly describe" question, which does not provide extra credit and only takes up additional time during the exam.
- Candidates should note that the sample answers provided in the examiner's report are not an exhaustive representation of all responses given credit during grading, but rather the most common correct responses.
- In cases where a given number of items were requested (e.g., "three reasons" or "two scenarios"), the examiner's report often provides more sample answers than the requested number. The additional responses are provided for educational value, and would not have resulted in any additional credit for candidates who provided more than the requested number of responses. Candidates are reminded that, per the instructions to the exam, when a specific number of items is requested, only the items adding up to that number will be graded (i.e., if two items are requested and three are provided, only the first two are graded).
- It should be noted that all exam questions have been written and graded based on information included in materials that have been directly referenced in the official syllabus, which is located on the CAS website. The CAS takes no responsibility for the content of supplementary study materials and/or manuals produced by outside corporations and/or individuals which are not directly referenced in the official syllabus.

EXAM STATISTICS:

- Number of Candidates: 596
- Available Points: 60.500
- Passing Score: 43.250
- Number of Passing Candidates: 312
- Raw Pass Ratio: 52.35%
- Effective Pass Ratio: 54.07%

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 1	
TOTAL POINT VALUE: 11	**LEARNING OBJECTIVE(S): A1, A2, A3, A5, A6, A7, A9**
SAMPLE ANSWERS	
Part a: 4.25 points	

Solution 1:
E(return) = risk-free +beta *(E(market) – risk-free)
E(X) = 12.2% (given)
E(Y) = 15.8% = 5% + 1.2 * (14% - 5%)
E(Z) = 13.1% = 5% + 0.9 * (14% - 5%)

Alpha = Forecast return – E(return)
A(X) = 15.0% - 12.2% = 2.8%
A(Y) = 18.0% - 15.8% = 2.2% (also given)
A(Z) = 12.5% - 13.1% = -0.6%

Sharpe = (Forecast return – risk-free) / std. deviation
S(X) = (15% - 5%) / 25% = 0.4 (also given)
S(Y) = (18% - 5%) / 20% = 0.65
S(Z) = (12.5% - 5%) / 30% = 0.25 (also given)
S(M) = 14% - 5%) / 15% = 0.60

Single stock recommendation – For a single stock, the total standard deviation is important. Stock Y is recommended due to its Sharpe ratio being higher than that for stocks X and Z. Other acceptable response included Stock Y is recommended because it has the highest forecasted (or expected) return and lower standard deviation than either stocks X or Z.

Comparison of the single stock recommendation to the market – stock Y has a larger Sharpe value than the market. Therefore, its risk-reward trade-off is a better alternative than investing in the market index.

Addition to a well-diversified portfolio – for an addition to an already well-diversified portfolio, the stock with the highest alpha is the key criteria. Stock X's alpha is higher than that for stocks Y and Z. Therefore, stock X is the recommended addition.

Solution 2

For the investor holding a single stock portfolio, I would recommend choosing Stock Y, because stock Y has the largest forecast return and smallest standard deviation. According (to) mean-variance criteria, stock Y is the best. For an investor considering adding one of these stocks to an already well-diversified portfolio, I would seek the stock which provide the largest alpha according to CAPM.
α_X= 15% - (rf + β_mx (E(Rm) – rf) = 15% - (5% + 0.8 x (14%-5%)) = 2.8%
α_Y= 18% - (5% + 1.2 x 9%) =2.2%
α_Z= 12.5% - (5% +0.9 x 9%) = -0.6%

Therefore, stock X would be recommended to the investor to adding to the well-diversified portfolio.
The sharpe ratio for stock Y = (18% - 5%)/0.2 = .65
The sharpe ratio for the market index = (14% - 5%)/0.15 = 0.60 < 0.65
Stock Y is a better alternative.
Sample solution #2:
Calculate alphas:
X: 0.15 – (0.05 + 0.09 x 0.8) =.028
Y: 0.18 – (1.2 x 0.09 - 0.05) = 0.022
Z: 0.125 – (0.09 x 0.09 -.05) = -0.006
Calculate Sharpe Ratios:
X: 0.1/0.25 = 0.4
Y: 0.13/0.2 = 0.65
Z: 0.075/0.3 = 0.25
Market: 0.09/0.15 = 0.6
For an investor holding a single stock, I would recommend stock Y. Stock Y has the highest Sharpe ratio (0.65). This risk reward trade-off is also better than investing in the market because 0.65 > 0.6.
For an investor adding to a portfolio, I would recommend stock X. It has the highest alpha (.028) which means it obtains a return higher than that suggested by CAPM. Additionally, it has the lowest Beta (.8). This suggests that it is likely less correlated with the current portfolio and will provide a diversification benefit.

Part b: 2.25 points

Sample 1

CAPM = $r_f + \beta(r_m - r_f)$

X: 0.05 + 0.8(0.14-0.05) = 0.122
Y: 0.05 + 1.2(0.14-0.05) = 0.158 (CAPM Returns)
Z: 0.05 + 0.9(0.14-0.05) = 0.131

$U = E(r) - \frac{1}{2}A\sigma^2$

Stock X: $0.122 - \frac{1}{2}(4)(0.25)^2 = -0.003$
Stock Y: $0.158 - \frac{1}{2}(4)(0.2)^2 = 0.078$
Stock Z: $0.131 - \frac{1}{2}A(0.3^2) = -0.049$ A = 4

Risk Averse: Stock Y has highest utility based on CAPM.
Risk Neutral (A=0): Stock Y also has highest utility (0.158)

Sample 2

CAPM x = 0.122
CAPM y = $E(r) = r_f + B_y(E(r_m) - r_f) = .158$
CAPM z = .05 + .9(.14-.05) = .131

$U_z = -0.049 = 0.131 - 0.5\ A\ (.3)^2$ A=4
$U_x = -0.003$

$U_y = 0.078$

Risk averse would choose stock Y as it has the lowest standard deviation with the highest return. Risk neutral would also choose Y as it has the highest return regardless of standard deviation.

Sample 3

$E(r_x) = .05 + .8(.14-.05) = .122$
$E(r_y) = .05 + 1.2(.14-.05) = .158$
$E(r_z) = .05 + .9(.14-.05) = .131$

Utility Z $-.049 = .131 - ½A(.3)^2$
$A=4$
Utility X: $.122 - ½(4)(.25)^2 = -.003$
Utility Y: $.158 - ½(4)(.2)^2 = .078$
Risk neutral A=0 and U=E(r) so choose stock with greatest return which is Y

Risk averse, suppose A=10
$U = E(r) - ½A\sigma^2$
X: $.122 - 5(.25)^2 = -.1905$
Y: $.158 - 5(.2)^2 = -.042$ So choose Y
Z: $.131 - 5(.3)^2 = -.319$

Sample 4

X: .05 + .8(.14-.05) = .122
Y: .05 + 1.2(.14-.05) = .158 (CAPM Returns)
Z: .05 + .9(.14-.05) = .131

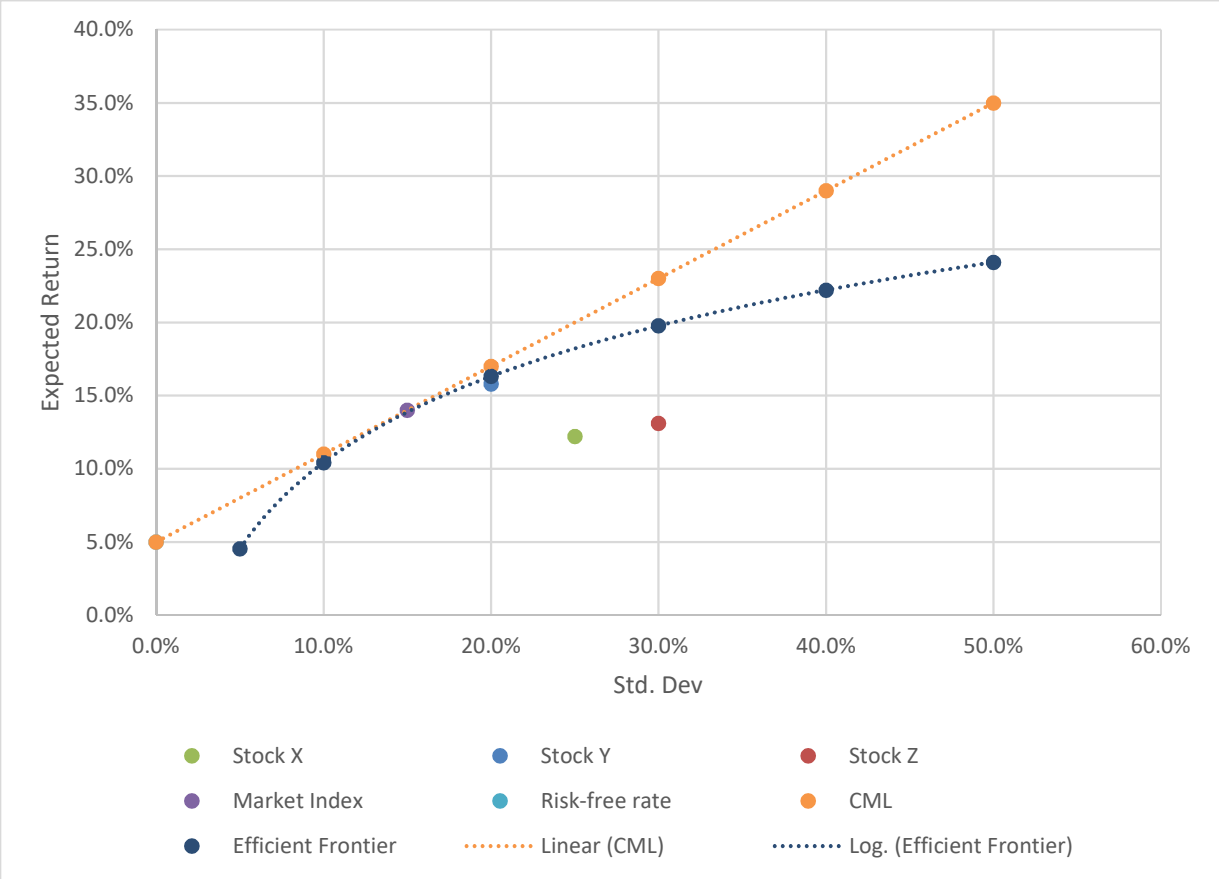
$U = E(r) - ½A\sigma^2$

Stock X: $.122 - ½(4)(.25)^2 = -.003$
Stock Y: $.158 - ½(4)(.2)^2 = .078$
Stock Z: $.131 - ½A(.3^2) = -.049$ A = 4

Risk neutral investor doesn't care about risk so choose Y since it has the highest expected return. Risk averse investor wants reward for taking on risk, so choose the stock with the best risk-reward tradeoff. Choose Y because it has the best Sharpe ratio (from Part a.)

Part c: 2 points

Graph:



X on EF

Accepted answers included:

- X is not on the Efficient Frontier because it is dominated by Y
- X is not on the Efficient Frontier because it is dominated by the market index
- Single stocks, such as X, are not on the efficient frontier, only well-diversified portfolios are
- X has diversifiable risk so it cannot lie on the efficient frontier

Y in portfolio

Accepted answers included:

- The investor will see Y as underpriced due to its forecast return being greater than its CAPM expected return.
- The investor will see Y as underpriced due to its positive alpha.
- The investor will see Y as underpriced due to it being above the SML

Part d: 2.5 points

Sample 1

E(X) = 12.2% σ_X = 25%

E(Y) = 15.8% σ_Y = 20%

Cov(X,Y) = $\rho\sigma_X\sigma_Y$ = (.60)(.25)(.20) = .030

Covariance Matrix

	X	Y
X	.0625	.030
Y	.030	.040

min W(X) = [σ_Y^2 – Cov(X,Y)] / [σ_X^2 + σ_Y^2 – 2 Cov(X,Y)]
[.040 - .030]/[.0625 + .040 – 2(.030)] = .235
min W(Y) = 1 – min W(X) = .765

Standard deviation of two-stock portfolio:
= [W(X)^2 x σ_x^2 + W(Y)^2 x σ_y^2 + 2 x W(X) x W(Y) x Cov(X,Y)]^.5
= [(.235)^2 x .0625 + (.765)^2 x .040 + 2(.235)(.765)(.03)]^.5
= .194 or 19.4%
The minimum-variance portfolio has a standard deviation smaller than either of the individual component stocks.

Sample 2 – Alternative formula for Covariance
E(X) = 12.2% σ_X = 25%
E(Y) = 15.8% σ_Y = 20%

Cov(X,Y) = β_X β_Y σ_M = (0.8)(1.2)(.15) = .0216

Covariance Matrix

	X	Y
X	.0625	.0216
Y	.0216	.040

min W(X) = [σ_Y^2 – Cov(X,Y)] / [σ_X^2 + σ_Y^2 – 2 Cov(X,Y)]
[.040 - .0216]/[.0625 + .040 – 2(.0216)] = .310
min W(Y) = 1 – min W(X) = .690

Variance of two-stock portfolio:
= W(X)^2 x σ_x^2 + W(Y)^2 x σ_y^2 + 2 x W(X) x W(Y) x Cov(X,Y)
= (.310)^2 x .0625 + (.690)^2 x .040 + 2(.310)(.690)(.0216)
= 0.034 or 3.4%

The Insurance Principle of Diversification states that by combining two less than perfectly correlated risks while keeping the total portfolio size constant, the overall risk is reduced. This is demonstrated by the variance of the portfolio (0.034) being less than the variance of X (0.0625) or Y (0.04) alone, so diversification is achieved.

EXAMINER'S REPORT
Candidates are expected to understand key concepts around risk appetite, tolerance and risk aversion and apply those to select a portfolio from many alternative portfolios using investor's

preferences such as utility function. Candidates are also expected to demonstrate knowledge of concepts such as Markowitz Portfolio Selection Model, the CAPM model, single factor model, zero beta portfolio.

Part a

Candidates were asked to recommend which stock an investor should select for:

a. A single stock portfolio, and
b. As an addition to an already well-diversified portfolio.

In addition, candidates were asked to demonstrate that the risk-reward trade-off for the recommended single stock is a better alternative than investing in the market index.

As such, candidates were expected to know that the relevant risk measure for a single stock is standard deviation; in such situations, the stock's beta is irrelevant and the appropriate criteria is overall risk and return – the stock with the highest Sharpe ratio. Comparison of Sharpe ratios for the three stock reveals stock Y's to be the highest. Comparison of Sharpe ratios for stock Y and the market again shows stock Y's to be the highest.

Candidates were expected to know that the relevant measure for adding a single stock to an already well-diversified portfolio is the stock's alpha. Candidates were expected to know that the alpha of a stock is its expected return excess of that required by the CAPM. Therefore, stock X with an alpha of 2.8% is the recommendation.

Common mistakes include:

- Providing the incorrect evaluation criteria for the different scenarios
- Failing to provide a recommendation
- Failing to demonstrate why the single stock selection was a better alternative than the market index.
- Calculation errors around expected returns, alpha and Sharpe ratios.
- Using utility theory approach to recommend stock selections

Part b

Candidates were expected to use CAPM returns to calculate utilities for each stock. They were expected to recognize that they had enough information about the utility of stock Z to be able to calculate the investors risk aversion factor (A). Candidates were expected to recognize that since the calculated A was greater than zero, the investor was risk averse.

Candidates were expected to use the calculated utilities to select Stock Y for the risk averse scenario.

Candidates were expected to select Stock Y for the risk neutral scenario by observing that Stock Y has the highest return and that this is all the risk neutral investor cares about since they are not concerned with risk.

Common errors include:

- Using returns other than CAPM expected returns in the Utility calculations.
- Not including any utility calculations anywhere in the response.
- Not recognizing that for risk averse investors, A>0 and that for risk neutral investors A=0.

Part c

Graph

Candidates were expected to draw a graph that included:

- The axes labeled correctly as expected risk and return
- The CML drawn with intersection of the y-axis at the risk-free rate
- The market return, correctly placed on the graph and on the CML
- The stocks graphed according to their expected CAPM
- The efficient frontier drawn as the top half of a parabola with the market index on the frontier

Most candidates were able to receive credit for much of the question.

Common mistakes included:

- Plotting the stocks according to their forecast return rather than their expected return (VERY common).
- Drawing the efficient frontier, a full parabola of the investible universe without specifying that the top half is efficient (VERY common).
- Beta-return graphs drawn rather than risk-return graphs.
- Market index not hitting the efficient frontier

X on EF

Candidates were expected to state a specific stock/portfolio that had better return for the given standard deviation or that diversifiable risk cannot be present on the efficient frontier.

Most candidates answered this part correctly.

Common mistakes included:

- Explaining X's relationship to the Efficient Frontier by "a stock/portfolio of stock's/etc. can be created that has a greater return but lower standard deviation" (needed to specify the stock or portfolio)
- Explaining X's relationship to the Efficient Frontier via the Sharpe ratio
- Explaining X's relationship to the Efficient Frontier via the alpha value.

Y in portfolio

Candidates were expected to specify whether Y was over/underpriced as well as explain why to receive full credit.

Common mistakes included:

- Answering by comparing Y to the other stocks or the market (partial credit).
- Answering it is underpriced as indicated by it lying above the CML.

Part d

Candidates were expected to correctly calculate the covariance between the two dominant stocks (X and Y) in the problem, use this covariance to calculate the weights of stocks X and Y that would create a portfolio with minimum variance, and then calculate the standard deviation of this constructed minimum variance portfolio.
Then, candidates needed to compare this calculated portfolio standard deviation to the standard deviation of the individual stocks in order to demonstrate that the Insurance Principle of Diversification was achieved..

Common errors made by candidates include:

- Using the formula which produces the optimal risky portfolio instead of the minimum variance portfolio to calculate the weight in each stock
- Comparing only the Sharpe Ratio of the minimum variance portfolio to the Sharpe Ratios of the individual stocks, which does not demonstrate diversification itself
- Failing to compare the portfolio's standard deviation to either of the individual component stocks, or being too general in the description of "reduced risk with the portfolio" without indicating a risk measure that can be used to quantify that reduced risk (that is, the standard deviation)

Comparing the minimum variance portfolio's standard deviation to the standard deviation of only Stock X (a comparison to only the standard deviation of Stock Y was given credit, since this stock has the lowest standard deviation of the two)

QUESTION 2	
TOTAL POINT VALUE: 7.25	**LEARNING OBJECTIVE(S): A.6., B.2., B.4., D.1., D.2., D.3., D.5.**

SAMPLE ANSWERS

Maximum Premium Allowed by Regulator

Determine risk-free rate from bond price:

$$1048.90 = \frac{40}{(1+r_f)^1} + \frac{1040}{(1+r_f)^2}$$

$$1048.90(1+r_f)^2 - 40(1+r_f) - 1040 = 0$$

$$1 + r_f = \frac{40 \pm \sqrt{40^2 - 4(1048.90)(-1040)}}{2(1048.90)}$$

$$r_f = .015 = 1.5\%$$

Solve for maximum premium allowed by regulator:

$$PV(Underwriting\ Cash\ Flows) \leq 7\%\ of\ Premium$$

$$(P - 39) - \frac{75(.50)}{1.015} - \frac{75(.30)}{1.015^2} - \frac{75(.20)}{1.015^3} \leq .07P$$

$$\boldsymbol{P \leq 120.57\ M}$$

Minimum Premium Required by Shareholders

Solution 1:

	T = 0	T = 1	T = 2	T = 3
Premium	P			
Expense	39			
Paid Loss		37.5	22.5	15
Reserves	75	37.5	15	
Surplus	37.5	18.75	7.5	
Contribution	151.5-P			
Assets Pre-Distribution	112.5	81.19	36.84	8.74
Distribution		24.94	14.34	8.74
Assets Post-Distribution	112.5	56.25	22.5	

IRR required:

$$IRR = r_f + \beta[(E(r_m) - r_f]$$

$$IRR = .015 + 1.5[.085 - .015]$$

$$IRR = .012 = 12\%$$

NPV(Equity Flows) >= 0

$$(P - 151.5) + \frac{24.94}{1.12} + \frac{14.34}{1.12^2} + \frac{8.74}{1.12^3} \geq 0$$

$$\boldsymbol{P \geq 111.58\ M}$$

Solution 2:

	T= 0	T = 1	T = 2	T = 3
Premium	P			
Expense	-39			
Losses		-37.5	-22.5	-15
Reserves	75	37.5	15	
Surplus	37.5	18.75	7.5	
Total Assets	112.5	56.25	22.5	
Invest Income		6.1875	3.09375	1.2375
UW Income	P – 114			
Change in Surplus	-37.5	18.75	11.25	7.5
Net Cash Flow	P-151.5	24.9375	14.34375	8.7375

IRR required:

$$IRR = r_f + \beta[(E(r_m) - r_f]$$

$$IRR = .015 + 1.5[.085 - .015]$$

$$IRR = .012 = 12\%$$

NPV(Equity Flows) >= 0

$$0 = P - 151.5 + \frac{24.9375}{1.12} + \frac{14.34375}{1.12^2} + \frac{8.7375}{1.12^3} \geq 0$$

$$\boldsymbol{P = 111.58\ M}$$

Premium to Satisfy Product Manager's Request

Solution 1:

$$Surplus\ at\ steady\ state = 37.5 + 18.75 + 7.5 = 63.75$$

$$\frac{T}{S} = \frac{I}{A}\left(1 + \frac{R}{S}\right) + \frac{U}{S}$$

$$30\% = 5.5\% \ (1 + 2.0) + \frac{U}{63.75}$$

$$U = 8.61 \ M$$

$$P = L + E + U$$

$$P = 75 + 39 + 8.61$$

$$\boldsymbol{P = 122.61 \ M}$$

Solution 2:

$$Total \ CY \ ROS = \frac{CY \ UW \ Income + CY \ Investment \ Income}{CY \ Surplus}$$

$$CY \ UW \ Income = P - L - E$$

$$CY \ UW \ Income = P - 75 - 39 = \ P - 114$$

$$CY \ Reserves = 75 + 37.5 + 15 = 127.5$$

$$CY \ Surplus = \ 37.5 + 18.75 + 7.5 = 63.75$$

$$CY \ Investment \ Income = (127.5 + 63.75)(.055) = 10.1875$$

$$30\% = \frac{P - 114 + 10.1875}{63.75}$$

$$\boldsymbol{P = 122.61 \ M}$$

Solution 3:

$$ROE = \frac{\sum_{j=1}^{n} I_j (1+g)^{-(j-1)}}{\sum_{j=0}^{n-1} Q_j (1+g)^{-j}}$$

$$g = 0 \ (steady \ state)$$

Incomes at steady state (CY 3 and beyond):

$$UW \ Income \ in \ CY3 \ from \ PY \ 3 = \ P - 75 - 39$$

$$Investment \ Income \ in \ CY3 \ from \ PY \ 1 = 1.2375$$
$$Investment \ Income \ in \ CY3 \ from \ PY \ 2 = 3.09375$$
$$Investment \ Income \ in \ CY3 \ from \ PY \ 3 = 6.1875$$

$$CY3\ Surplus = 37.5 + 18.75 + 7.5 = 63.75$$

$$30\% = \frac{(P - 114) + 1.2375 + 3.09375 + 6.1875}{63.75}$$

$$\boldsymbol{P = 122.61\ M}$$

Solution 4:

$$(P - L - E) + (S + P - E)y = kS$$

$$(P - L - E) + (R + S)y = kS$$

$$Steady\ State\ Reserves = 75 + 37.5 + 15 = 127.5$$

$$Steady\ State\ Surplus = 37.5 + 18.75 + 7.5 = 63.75$$

$$(P - 75 - 39) + (127.5 + 63.75)(.055) = (0.30)(63.75)$$

$$\boldsymbol{P = 122.61\ M}$$

Grader's Note: Candidates who used this approach were expected to use the investment yield of 5.5% that was given rather than the risk-free rate.

The solution above reflects replacing "S+P-E" (investable assets in the formula) with "R+S" (steady-state investable assets as given). Ideally, candidates would have made this adjustment, but were not necessarily expected to recognize this difference and make this adjustment under exam conditions.

Solution 5:

$$(P - L - E) + (S + P - E)y = kS$$

$$P = \frac{S(k - y) + L}{1 + y} + E$$

$$Steady\ State\ Reserves = 75 + 37.5 + 15 = 127.5$$

$$Steady\ State\ Surplus = 37.5 + 18.75 + 7.5 = 63.75$$

$$P = \frac{63.75(0.30 - .055) + 75}{1 + .055} + 39$$

$$\boldsymbol{P = 124.89\ M}$$

Grader's Note: Candidates who used this approach were expected to use the investment yield of 5.5% that was given rather than the risk-free rate. This solution does not make the adjustment to investable assets as described in solution 4 above.

Recommended Premium

Example 1
Recommend charge 120.57, this is the requirement of regulator. Underwriting cashflow (present value) does not exceed 7% of premium; also meet investor's requirement of having IRR > 12%. Slightly missed the PM's request though.

Example 2
The premium must be less than 120.57 according to the regulator, greater than 111.58 according to investors, and greater than 122.61 according to product manager. To satisfy the most <illegible word>, charge the maximum regulator allowed premium of 120.57. This doesn't quite satisfy the product manager's 122.61 request, but it's the closest allowed. It also exceeds shareholder requirements.

Example 3
Recommendation: Charge P = 120.57
This satisfies regulators and shareholders, & comes very close to satisfying the product mgr.

Example 4
I recommend $120.56 = P which satisfies investors & regulators. Product manager would NOT be completely satisfied but its better, probably, than not filing

Example 5
The premium demanded by the Product Manager will result in a rate that will be declined by the regulator. However, the rate required by the shareholder will result in a premium less than what is required by the regulator. I would decide to charge 120,570,384 which will earn a return greater than investors require and still satisfy regulators.

EXAMINER'S REPORT

The question was constructed from the point of view of a pricing actuary. Candidates were presented with information about the company and the market. Two "requirements" were provided (regulator perspective and shareholder perspective). An additional "request" was also provided (management perspective).

Candidates were asked to "recommend a premium" and describe how the recommended premium related to "the premiums needed to satisfy" the regulators, shareholders, and the Product Manager.

Candidates were expected to calculate the "premiums needed to satisfy" each of the three parties.

Candidates were expected to use the "maximum regulator premium" and "minimum shareholder premium" (presented as "requirements") to bound the range of acceptable premium levels. Candidates

were then expected to use the Product Manager premium (presented not as a "requirement", but as a "request") to help decide what premium to charge within the acceptable bounds.

Maximum Premium Allowed by Regulator

Candidates were expected to:

- Determine the risk-free rate from the bond price
- Correctly identify UW Cash Flows (premiums, expenses and losses) and discount to time 0
- Use the risk-free rate to discount losses
    - Because the requirement was prescribed by the *regulator*, we expected candidates to recognize to use the risk-free rate
- Set up the equation correctly:

$$P - E - \frac{Paid\ Loss_1}{1 + r_f} + \frac{Paid\ Loss_2}{(1 + r_f)^2} + \frac{Paid\ Loss_3}{(1 + r_f)^3} = .07P$$

Common mistakes included:

- Calculation errors in determining the risk-free rate
- Not correctly identifying UW cash flows
- Confusing UW cash flows with UW income
- Including investment income with UW cash flows
- Not determining the "Present Value" (i.e. discounting to a time other than T = 0)
- Not setting the PV(UW cash flows) equal to 7% of premium
- Using a 12% discount rate
- Using a 5.5% discount rate
- Using an 8.5% discount rate
- Not actually solving for the maximum premium allowed by regulators
- Calculation errors in solving for premium

Minimum Premium Required by Shareholders

Candidates were expected to:

- Determine the required IRR from the CAPM formula
- Correctly identify Loss Reserves, Surplus, Investable Assets and Investment Income at each time
- Correctly calculate Equity Flows at each time. This could be calculated as either:

$$EQFlow_t = UW\ INC_t + INV\ INC_t - \Delta S_t$$

$$EQFlow_t = (IA_{t-1})1.055 + UWCF_t - IA_t$$

- Set the net present value of the equity flows greater than or equal to 0 using the calculated IRR rate

Common mistakes included:

- Calculation errors in determining Reserves, Surplus, Assets or Equity Flows
- Deconstructing the company beta into components
- Assuming a different investment income rate on surplus versus reserves
- Incorrectly calculating the Equity Flow
- Sign errors when setting up the NPV calculation
- Setting Investable Assets equal to Surplus
- Applying the Steady-State assumption to the IRR Framework
- Using a discount rate other than the calculated IRR rate
- Attempting to use a method other than IRR
- Not actually solving for the minimum premium required by Shareholders
- Calculation errors in solving for premium

Premium to Satisfy Product Manager's Request

Candidates were expected to calculate the requested premium based on the "total steady-state calendar year return on surplus" criterion. Candidates were expected to understand what was meant by "total steady-state calendar year return on surplus," calculate the steady-state surplus held by the company, identify calendar year underwriting and investment income, and solve for the premium required to achieve the target return.

Common mistakes included:

- Using a method that is inconsistent with a steady-state calendar year return on surplus and the information provided, such as the internal rate of return (IRR) or PVI/PVE methods, or the Growth Model with positive growth.
- Discounting of income and/or surplus
- Using only one year of reserves and/or surplus instead of calculating the values in a steady-state
- Not reflecting investment income
- Using an incorrect investment yield
- Using incorrect investable assets or otherwise only partially accounting for investment income
- Not calculating a premium

Recommended Premium

Candidates were expected to recommend a premium to charge, based on the results as calculated for the regulator, shareholder, and product manager. Any errors candidates may have made in calculating the three premiums were considered while grading the response.

Common mistakes included:

- Misinterpretation of the directional constraints of each of the stakeholders
- Not considering the viewpoint of one or more stakeholders in determining the premium recommendation
- Failure to make a clear premium recommendation
    - Calculating two or more of the premium requirements but failing to indicate which, if any, represented the recommendation.

- Recommending a range of premium values rather than a single premium
- Misapplication of the priority of the stakeholders' requirements; for example, misunderstanding that the product manager's criterion was a request rather than a profit requirement.

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 3	
TOTAL POINT VALUE: 2.75	**LEARNING OBJECTIVE(S): A6, A7**

SAMPLE ANSWERS

Part a: 1.5 points

Sample 1

E(r) = .05 +1.115(.11-.05) =.1169

1.5 = (E(r_M) – .05) / .20^2 → E(r_M) =.11

$\beta_{portfolio}$= .20(1.10) + .50(1.25) +.3(0.90) = 1.115

Sample 2

Market price of risk = 1.5 = (r_M - .05) / .2^2 = (r_M – r_f)/ σ_M^2

R_M =.11=11%

E(r) = r_f + β * (r_M – r_f)

A = .05+1.1(.06) =.116

B .05+1.25(.06) =.125

C .05+.9(.06) =.104

Rate on portfolio = 20(.116) + 50(.125) + 30 (.104) / 100 =11.69%

Part b: 0.75 point

Sample 1:

Zero-beta CAPM is used when borrowing is restricted. Construct the zero-beta portfolio on the underside of the efficient frontier & use in place of the risk-free asset.

Sample 2:

This incorporates fact that investors can't borrow at risk-free rate. Zero-beta portfolio is uncorrelated with original portfolio on opposite side of the efficient frontier. Investors will invest in higher beta stocks: zero-beta CAPM adjusts expected return.

Sample 3:

Zero Beta version of CAPM is considered when an investor faces borrowing restrictions; they cannot borrow at risk free rate. Instead the expected return of the zero-beta portfolio is used in place of the risk-free rate. The zero-beta portfolio lies on the inefficient side of the minimum variance frontier and is not correlated with market portfolio.

Sample 4:

Zero-beta CAPM

-is CAPM with borrowing restrictions

-for every portfolio on the efficient frontier there is a portfolio on the opposite side (inefficient side) that is uncorrelated with the portfolio on the efficient side that is a zero-beta portfolio

-zero-beta CAPM has a lower Sharpe-ratio and thus less reward for taking risks than traditional CAPM.

Sample 5:

CAPM assumes all can borrow at risk-free rate without restrictions. Zero-beta version breaks that assumption and uses a rate higher than r_f instead, to better reflect reality that all cannot borrow at risk-free rate, without limits and in all situations. The SML will be flatter than in the regular CAPM.

Sample 6:
The zero-beta formula is used when it's not possible to invest in the risk-free asset
$E(r_i) = E(r_{\text{zero beta portfolio}}) + \beta_i [E(r_M) - E(r_{\text{zero beta portfolio}})]$
A zero-beta portfolio is a portfolio on the inefficient side of the frontier that is uncorrelated with the market portfolio.

Part c: 0.5 point

Sample 1:
E(r) = .08+1.115(.11-.08) =.11245=11.345%

Sample 2:
$r_i = r_z + B\_i(r_M - r_z)$
A .113
B .1175
C .107

$R_p = \sum w_{i*} r_i = 11.345\%$

EXAMINER'S REPORT

Candidates were expected to understand that the market price of risk = market risk premium / market variance. Candidates were expected to demonstrate an understanding of CAPM and how to use the capitalizations provided to calculate either the weighted average return or the weighted average beta. Candidates were expected to understand the zero-beta CAPM and that the risk-free rate in the CAPM SML is replaced by the expected return on the zero-beta portfolio.

Part a

- Candidates were expected to
  - derive the market risk premium or the market expected return
  - calculate the weighted average beta using the provided capitalization for each stock and then use this beta, the given risk-free rate and the calculated market risk premium to calculate the portfolio expected return OR
  - calculate the expected return for each stock using the given risk-free rate and the given beta as well as the calculated market risk premium. Then candidates needed to calculate the portfolio's expected return using the weighted average of the returns based on the given stock capitalizations.
- Common mistakes include:
  - not using the correct market price of risk ratio.

- providing a final answer in dollars instead of providing the percentage expected return (i.e. 11,169 instead of 11.169%).
- using the formula of sharp ratio instead of the correct formula for market price of risk ratio.

Part b

- Candidates were expected to explain the zero-beta CAPM.
- Common mistakes include:
  - Stating that the zero-beta portfolio has the same risk as the given risky portfolio did not receive credit unless the candidate explicitly stated that the zero-beta portfolio has the same risk as the corresponding risky portfolio in a situation where the minimum variance frontier is symmetric around the global minimum variance portfolio.
  - Stating that zero beta portfolio is a stock on the inefficient frontier. The zero-beta portfolio is a portfolio of stocks constructed to have a beta of zero.
  - Stating that zero-beta portfolio is negatively correlated with the market portfolio instead of stating that it is uncorrelated with the market portfolio.
  - Stating that zero-beta portfolio is uncorrelated to the entire efficient frontier
    Stating that there are short-selling restrictions without specifying borrowing restrictions at risk-free rate.

Part c

Candidates were expected to replace the risk-free rate with the given expected return on the zero-beta portfolio.

Common mistakes include:

- Recalculating the market risk premium based on incorrect assumption that the given market price of risk would be equal to $(E(r_M) - E(r_z)) / \sigma_M^2$.
- Recalculating $E(r_z)$ based on incorrect assumption that the given market price of risk would be equal to $(E(r_M) - E(r_z)) / \sigma_M^2$.

Using value of $E(R_M)$ derived in part a in part c calculation for $E(r_M)$.

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 4	
TOTAL POINT VALUE: 3.75	**LEARNING OBJECTIVE(S): B3**
SAMPLE ANSWERS	

Part a: 2 points

Sample 1

A sold at par → r = 6%

18,708 / 1.06 ^ 1.5 = 17,142
10,226 / 1.06 ^ 3.5 = 8,339
PV(L) = 25,481
D(L) = (1.5 x 17,142 + 3.5 x 8,339) / (17,142 + 8,339) = 2.155

D(S. A) = [(1 x 6) / 1.06 + (2 x 106) / 1.06^2] / [6 / 1.06 + 106 / 1.06^2] = 1.943

D(S. B) = 4

1.943 x W(A) + (1 – W(A)) x 4 = 2.155
W(A) = 0.897
W(B) = 0.103

Invest A: 25,481 x 0.897 = 22,856
Invest B: 25,481 x 0.103 = 2,625

Sample 2

Duration of Obligations:

Time	Amt	PV	PV x t
1.5	18,708	18,708 / 1.06^1.5 = 17,142.28	25,713.42
3.5	10,226	10,226 / 1.06^3.5 = 8339.41	29,187.94
Total		25,481.69	54,901.36

Note: Disc rate is 6% (coupon rate) as bond is selling at par. Yield = coupon rate.
Duration = 54,901.36 / 25,481.69 = 2.155

Duration of A:

Time	Amt	PV	PV x t
1	60	56.60	56.60
2	1,060	943.40	1,886.8
Total		1,000	1,943.4

Duration = 1,943.4 / 1,000 = 1.943

Duration of B = 4 since it's zero coupon bond.

Let *w* be the weight in A: set up duration matching.
$1.943w + 4(1 - w) = 2.155$

1.943w + 4 – 4w = 2.155
w = 89.69%

So invest 25,481.53 x w = 22,854.53 in Security A
25,481.53 x (1 – w) = 2,627.16 in Security B

Sample 3
PV(Liabilities) = 18,708 + 10,226 = 28,934 → Assume values are given at time zero
Duration Liab = [18,708 x 1.5 + 10,226 x 3.5] / 28,934 = 2.2 years

Duration A = [1 x 6 / 1.06 + 2 x 106 / 1.06^2] / 100 = 1.9434

Duration B = 4

w_A x 1.9434 + (1 – w_A) x 4 = 2.2
w_A = 0.875
w_B = 0.125

Security A: 0.875 x 28,934 = 25,317.25
Security B: 0.125 x 28,934 = 3,616.75

Sample 4
Duration (Bond selling at par) = $\frac{(1+y)}{y} \times \left(1 - \frac{1}{(1+y)^t}\right) = \frac{(1.06)}{0.06} \times \left(1 - \frac{1}{(1.06)^2}\right) = 1.943$
Duration(Zero coupon bond) = maturity = 4

Assume 18,708 and 10,226 are present value amounts.
Duration = $\frac{1.5\times 18.708+3.5\times 10,226}{18,708+10,226} = 2.207$

2.207 = w(par) x 1.943 + (1- w(par)) x 4
w(par) = 0.8719
0.8719 x (18,708 + 10,226) = 25,228 in par bond
(1 – 0.8719) x (18.708 + 10,226) = 3,706 in zero coupon bond

Part b: 1.25 points

Sample 1
Convexity = $\frac{1}{P\ (1+y)^2} \sum PVCF \times n \times (n+1)$

Time	1	2	3	4
PV CF A	22,856 x 0.06 / 1.06 = 1,294	22,856 x 1.06 / 1.06^2 = 21,562		
PV CF B				2,625

Convexity = $\frac{1}{25{,}481 \times 1.06^2}(1{,}294 \times 1 \times 2 + 21{,}562 \times 2 \times 3 + 2{,}625 \times 4 \times 5) = 6.443$

Sample 2

$P_A = 6\ (1+y)^{-1} + 106\ (1+y)^{-2}$
$dP_A/dy = -6\ (1+y)^{-2} - 212\ (1+y)^{-3}$
$d^2P_A/dy^2 = 12\ (1+y)^{-3} + 636\ (1+y)^{-4} = 513.847$ (y = 0.06)
Conv A = 513.847 / 100 = 5.1384

$P_B = 100\ (1+y)^{-4}$
$dP_B/dy = -400\ (1+y)^{-5}$
$d^2P_B/dy^2 = 2{,}000\ (1+y)^{-6}$
Conv B = $2{,}000\ (1+y)^{-6}$ / $100\ (1+y)^{-4} = 20\ (1+y)^{-2}$ = 17.8 (assume y = 6%)

Conv Portfolio = 0.875 x 5.1385 + 0.125 x 17.8 = 6.7211

Part c: 0.5 point

Sample 1

Since convexity of new portfolio is higher, an increase in rates would mean a smaller decrease in price. If rates decrease, price will increase more for the new one, so I would recommend the new one.

Sample 2

Recommend the alternative portfolio. 6.74 > 6.44 and the durations are the same; our portfolio is still immunized, but the greater convexity means we have a greater upside price change if yields fall (the downside price change is also smaller).

Sample 3

Convexity of investment should match convexity of liabilities ideally.

Convexity of Liabilities = $\frac{1.5 \times 2.5 \times \frac{18.708}{1.06^{1.5}} + 3.5 \times 4.5\ \frac{10{,}226}{1.06^{3.5}}}{25{,}482 \times 1.06^2} = 6.83$

Because convexity of liabilities is closer to new investment, new investment would be preferred.

EXAMINER'S REPORT

Candidates were expected to understand various strategies to manage interest rate risk in a bond portfolio, how to apply those strategies and how interest rate changes impact bond prices.

Part a

Candidates were expected to recognize the interest rate for coupon bonds selling at par is equivalent to the coupon rate. Candidates were expected to calculate the duration of the obligations, as well as the durations of Security A and Security B, and use those durations to determine an immunizing portfolio consisting of Security A and Security B.

The question did not state whether the obligation amounts were in present value or future value. Credit was given under either assumption.

Common errors included:

- Assuming the obligations were stated in present value (or future value) when calculating the duration of the obligations, but not assuming they were stated in present value (or future value) when calculating the amounts to allocate between the bond portfolios.
- Assuming an interest rate different than 6%.
- Failing to apply the correct duration formula to the obligations and the two bond portfolios.

Part b

Candidates were expected to calculate the convexity of the immunizing portfolio comprised of Security A and Security B from part a.

Common errors included:

- Calculating the convexity of the obligations instead of the immunizing portfolio.
- Not incorporating the weights between Security A and Security B of the immunizing portfolio to calculate the overall convexity.
- Assuming Security B's present value amount from part a. is a future cash flow.
- Using the bond price approximation formula or not knowing the convexity formula.

Part c

Candidates were expected to recommend the higher convexity portfolio and provide a justification that included a direct comparison of the impact of interest rate changes between the immunizing portfolio from part a. and the new portfolio from part c. Alternatively, candidates were expected to calculate the convexity of the obligations, recommend the portfolio option with convexity closest to the obligation convexity, and provide rationale for why the recommended portfolio will require less rebalancing when interest rates change.

Common errors included:

- Selecting the lower convexity portfolio with insufficient justification.
- Providing a comparison of the impact of interest rate changes between the increase in a bond and the decrease of the same bond without comparing the two portfolio options.
- Justification provided is too vague (i.e., better for interest rate risk, investors like higher convexity).
- Misstating the relationship between interest rate changes and the impact on bond prices (i.e. stating that bond prices increase as rates increase).
- Stating incorrect assumptions about high convexity (i.e. stating higher convexity provides better immunization; higher convexity provides better approximate changes in yield; or higher convexity is less sensitive).

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 5	
TOTAL POINT VALUE: 1.25	**LEARNING OBJECTIVE(S): A9**

SAMPLE ANSWERS

Part a: 0.5 point

Sample 1

Semi-Strong: Price reflects all publicly available information vs. Strong: reflect all information including insider information

Sample 2

Semi-strong form states that price of stock should reflect all information that is publicly available. Strong-form states that price of each stock will reflect all information about the firm, both public information and private information.

Part b: 0.75 point

Sample 1

If you have private information about a new project that would increase the firm's value, you would expect stock price to increase only once the project is publicly known under semi-strong. This means that under semi-strong you are expected to earn abnormal returns if you have access to the info about the project before it is publicly announced. You would not be expected to earn abnormal returns under strong-form because prices will already reflect the new project even prior to public announcement.

Sample 2

The management of the firm know about a new product offering that will increase future returns. The stock price does not reflect this info since it is not public info.

This supports the semi-strong form since all public info is included in stock price. This does not support strong form since that which is not public is not reflected.

Sample 3

Post earnings announcement, price drift according to positive/negative announcements. If strong EMH, price will reflect before the earnings announcement.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of various forms of the efficient market hypothesis and convey understanding via an example that demonstrates the differences between strong and semi-strong.

Part a

Candidates were expected to demonstrate understanding of the differences between the semi-strong EMH and the strong EMH.

A common mistake was to accurately describe the semi strong form (or the strong form) but inaccurately describe the other.

Part b

Candidates were expected to construct an example that demonstrates the differences between strong and semi-strong. The question asked candidates to include a discussion of how the scenario would lead to abnormal returns.

Common mistakes were:

- Describing an observed anomaly of the EMH (such as Small Firm in January Effect) without explanation as to why that supports the semi-strong EMH and not the strong EMH.
- Not demonstrating that the scenario given would support the semi-strong form but not the strong form.

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 6	
TOTAL POINT VALUE: 4	**LEARNING OBJECTIVE(S): A8**
SAMPLE ANSWERS	

Part a: 1.25 points

$15\% = r_f + 1\,x + 0.5\,y$
$7.5\% = r_f - 0.1\,x + 0.25\,y$
$13.5\% = r_f + 0.3\,x + 0.75\,y$

Solve algebraically →

$r_f = 6\%\,; x = 5\%\,; y = 8\%$

Part b: 2.75 points

Solution 1

$E(r_D) = 0.06 + 0.2\,(0.05) + 0.5\,(0.08)$
$= 0.11 < .12$ therefore the opportunity exists

Let w_A be the weight to invest in A; w_B be the weight in B; w_c be the weight in C

To neutralize beta,
$w_A + w_B + w_C = 1$
$w_A - 0.1\,w_B + 0.3\,wc = 0.2$
$0.5\,w_A + .25\,w_B + .75\,w_C = 0.5$

Solve for w_A, w_B and w_C
$w_A = 1/9$; $w_B = w_C = 4/9$

Construct portfolio P by investing 1/9 in A, 4/9 in B and 4/9 in C

$\beta_{px} = 1/9\,(1.0) + 4/9\,(-.1) + 4/9\,(.3) = .2$
$\beta_{py} = 1/9\,(.5) + 4/9\,(.25) + 4/9\,(.75) = .5$

$E(r_P) = 1/9\,(.15) + 4/9\,(.075) + 4/9\,(.135) = .11$

To return 10,000, investment needed = 10,000 / .01 = 1M

Short sell 1M in portfolio P and use the proceeds to invest in portfolio D so that
$B_x = -.2 + .2 = 0$
$B_y = -.5 + .5 = 0$
Net return = -.11 + .12 = .01

Solution 2

$E(r_D) = 0.06 + 0.2\ (0.05) + 0.5\ (0.08) = 11\%$ according to factor model ≠ 12%
$B_x = 0.2 = 0.3 \times 2/3$
$B_y = .5 = 0.75 \times 2/3$

Buy portfolio D, short sell (2/3 Portfolio C and 1/3 Risk Free)
Expected return = 12% - (2/3 * 13.5% + 1/3 * 6%) = 1%

In order to have a riskless return of 10,000
Invest 10,000 / 0.01 = 1M in D and short sell Portfolio C 1M * 2/3 = 666,667 and borrow risk free 1M * 1/3 = 333,333

Solution 3
$E(r_D) = 0.06 + 0.2\ (0.05) + 0.5\ (0.08) = 11\%$
$\alpha = .12 - .11 = .01$

An arbitrage opportunity exists if the net investment at time 0 is zero, there is no risk, and a profit is guaranteed.

Buy 3D, sell A and 4B

$B_x = 3(.2) – 4\ (-.1) – 1 = 0$
$B_y = 3\ (.5) – 4(.25) - .5 = 0$
No risk.

Need to use r_f to make net investment 0
Buy 3D , sell 1 A, sell 4 B, buy 2 r_f to create an arbitrage opportunity (let this = portfolio P)
10000 = profit = α (3x)
x = 333333 of portfolio P to guarantee a risk free profit of 10000

EXAMINER'S REPORT

Candidates are expected to demonstrate knowledge of the factor model and creating arbitrage opportunity with the use of portfolios provided.

Part a

Candidates are expected to provide the factor model equation and correctly solve for the 3 unknowns.

The most common error was making an algebraic/calculation mistake.

Part b

Candidates are expected to identify the arbitrage opportunity by recognizing the mismatch between the outputs from the factor model and expected return provided on portfolio D. After that, candidates should create one of the 3 combinations of portfolio A, B and C that neutralize

beta from portfolio D. Also, candidates will need to borrow and lend at risk free rate to ensure zero net investment with certain combinations. Lastly, candidates are expected to provide the dollar amount invested in the portfolio that returns $10000.

Common mistakes included:

- Attempting to trade β_x or β_y directly
- Failing to create an arbitrage strategy with net zero investment by not borrowing/lending a risk free asset

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 7

TOTAL POINT VALUE: 1.5	**LEARNING OBJECTIVE(S): A10**

SAMPLE ANSWERS

Part a: 0.5 point

Sample solution

Conservatism: Occurs when investors are slow to update their beliefs in response to new evidence. In the graph provided, the positive earnings results came out on March 16 and the stock price did not reach a new equilibrium level until several days later.

Part b: 1.0 point

Sample solutions (any two of the following)

- Fundamental Risk – The expected correction may not have occurred in time for the arbitrage to be profitable. The timing of the price correction may exceed the hedging time frame. Market may remain irrational longer than investor remains solvent.
- Implementation Limitations - There may be limitations to short-selling such as prohibitions on short selling, preventing an investor from taking advantage of this opportunity.
- Transaction Cost - Even if the profit opportunity occurs, due to the transaction cost, the investor may not be able to profit from it
- Model Risk – The models used to identify the opportunity may have been flawed
- Liquidity Limitations - There may be limited stocks to buy and those stocks may already be held by other investors that aren't willing to sell.

EXAMINER'S REPORT

Candidates were expected to understand behavioral finance and how it can impact market efficiency and impact a rational investor's decision making process.

Part a

Candidates were expected to interpret the example provided and to identify and briefly describe the underlying information processing error.

Common errors included:

- Failure to both identify and briefly describe the correct information processing error exhibited in the example.
- Confusing the 'Post-earning announcement drift' phenomenon as an information processing error. 'Post-earnings announcement drift' was not accepted on its own as a correct identifier of the error type; it needed to be coupled with an error such as conservatism.

Part b

Candidates were expected to both identify and briefly describe two limits to arbitrage.

Common errors included:

- Failure to both identify and briefly describe the limits to arbitrage.

- Incorrectly listing behavioral biases instead of limits to arbitrage (framing, mental accounting, regret avoidance, affect, etc.). Note that the question specifically asked candidates to list limitations that would have prevented a "rational" investor from profiting. Responses that would have prevented an irrational investor from profiting were not accepted.
- In some cases, candidates did not provide sufficiently distinct limitations to arbitrage.

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 8

TOTAL POINT VALUE: 2	**LEARNING OBJECTIVES: B1, B2**

SAMPLE ANSWERS

Part a: 0.5 point

Sample 1

Price = $90/1.07 + 1090/1.08^2 = 1018.61$

Part b: 0.25 point

Sample 1

$f_{1,2} = (1+r_2)^2/(1+r_1) - 1 = 1.08^2 / 1.07 - 1 = 9\%$

Sample 2

$f_2 = (1.08)^2 / 1.07 - 1 = 9.01\%$

Part c: 0.5 point

Sample 1

Expectation hypothesis states that forward rate = expected future short rate so price = 1090/1.09 = 1000 or 1090/1.0901 = 999.91

Sample 2

$P_2 = 1018.61(1.07) - 90 = 999.91$

Part d: 0.75 point

Sample 1

Under liquidity preference, forward rate = expected short rate + liquidity premium so expected short rate = 9% - 3% = 6%

Price = 1090/1.06 = 1028.30

Sample 2

$F_2 = \exp(r_2) + 3\% \rightarrow \exp(r_2) = 9.01\% - 3\% = 6.01\%$

Price at t=1 = 1090 / 1.0601 = 1028.20

EXAMINER'S REPORT

Candidates were expected to understand the expectations hypothesis and the liquidity preference theory, use a yield curve to derive relevant forward rates, and apply these concepts in calculating bond prices.

Part a

Candidates were expected to apply the correct rates in the given yield curve to the payment schedule provided, in order to calculate the issue price of the bond.

Common mistakes included:

- Using the rate of 8% to discount the $90 coupon paid at the end of the first year.
- Incorrect or missing exponent in the denominator, e.g. using 1.08 rather than 1.08^2 to discount the payments made at time 2.
- Using the second year forward rate to discount the payments made at the end of the second year.
- Failing to include one of the coupon payments.

Part b

Candidates were expected to use the yield curve provided to calculate the forward rate for the second year.

A common mistake was failing to subtract 1 from the discount factor. Rounding the resulting forward rate to 9% was acceptable.

Part c

Candidates were expected to apply the expectations hypothesis by using the forward rate from part b to calculate the expected price of the two-year bond at the beginning of the second year.

Common mistakes included:

- Calculating the price of a new two-year bond at the beginning of the second year, rather than the expected price of the original two-year bond at that time. Credit was not given to answers of the form $90/1.09 + 1090/1.09^2$, because this formula yields the correct answer coincidentally, since the forward rate (very nearly) equals the coupon rate.
- Using the incorrect rate to discount coupon and principal payments.

Part d

Candidates were expected to adjust the forward rate calculated in part b. to account for the liquidity premium, and to use this adjusted forward rate to calculate the expected price of the bond at the beginning of the second year.

Common mistakes included:

- Using the wrong rate, e.g. adjusting 8% instead of 9% for the liquidity premium.
- Adding the liquidity premium to the forward rate, i.e. using a rate of 9%+3% = 12% to discount the bond payments.
- Including additional coupon payments that would have been made in the first year.

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 9

TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): C3**

SAMPLE ANSWERS

Part a: 0.5 point

Sample answers include:

- Swap bond X for bond Y to avoid the depreciation in price of X in 5 years.
- Switch from higher to lower duration bonds since expect rates to increase. Lower duration bonds are less sensitive to rate changes.
- Expect rates to go up, so sell 15-yr bond and buy 5-yr bond.

Part b: 0.75 point

Sample Response 1:

At time 0:

Value of Bond X = 1000 / (1.04^15) = 555.26
Value of Bond Y = 1000 / (1.04^5) = 821.93
Total investment in Y = 555.26 + 821.93 = 1377.19

Rate Anticipation Swap: Sell X and buy Y.

At time 5:

Value of current portfolio of X+Y = 1000/(1.06^10) + 1000 = 1558.39
Value of new portfolio of only Y = 1377.19 x 1.04^5 = 1675.56
Additional assets = 1675.56 – 1558.39 = 117.17

Part c: 0.5 point

- Intermarket spread swap – current spread between corporate and treasury (2%) is larger than historical (1%). If current spread is too large, buy corporate bonds then wait for spread to decrease when Treasury yields increase.

- Substitution swap – swap two bonds with almost identical characteristics by swapping out the lower yield for higher yield. In this case, Z has same characteristics as Y in terms of maturity and credit rating, but higher yield, thus, swap Y for Z.

EXAMINER'S REPORT

Candidates were expected to understand active bond management strategies and determine the most appropriate strategy in a given scenario.

Part a

Candidates were expected to know the effect of interest changes on bond prices and whether to increase/decrease the duration of portfolio in response. Candidates were expected to provide the direction of the changes.

Common errors:

- Simply stating that the company should change the duration of the portfolio when interest rate change or stating that prices will change when interest rates change.
- Describing how to structure an interest rate swap.
- Sell Y buy Z.

Part b

Candidates were expected to be able to calculate the value of a portfolio of bonds at different times.

Common errors:

- Mistaking $1000 in par value to mean $1000 at time 0.
- Calculating the loss in value of X if Treasury rate increase at time 5, instead of calculating the difference between the current and new portfolio at time 5.
- Assuming interest rate increase at time 0 instead of 5.

Part c

Candidates were expected to know the potential sources of mispricing in the context of active bond portfolio management and the type of bond swap that addresses it. Candidates were expected to identify the mispricing in the question in order to describe the best strategy and action to take.

Candidates were not expected to know the name of the type of swap (i.e. intermarket spread swap or substitution swap) but were expected to be able to describe its key features.

Common errors:

- Simply stating buy bond Z because it has a higher yield.
- Citing pure yield pickup swap or tax swap.
- Describing the conditions for arbitrage.

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 10	
TOTAL POINT VALUE: 1.5	**LEARNING OBJECTIVE(S): C1, C2**
SAMPLE ANSWERS	
Part a: 1 point	
Coverage Ratio (or times-interest earned ratio) =EBIT/Interest expense Company A: $4M/$2M = 2.0 Company B: $7M/$5M = 1.4 ***Return on Assets*** =EBIT/Assets Company A: $4M/$10M = 40% Company B: $7M/$25M = 28% Company A is less likely to default because they have a higher coverage ratio and a higher return on assets ratio. **Other Acceptable Ratios:** ***Return on Assets Net of Interest Expense*** =(EBIT – Interest Expense)/Assets Company A: ($4M - $2M)/$10M = 20% Company B: ($7M - $5M)/$25M = 8% ***Return on Sales (Profit Margin)*** =EBIT/Revenue Company A: $4M/$20M = 20% Company B: $7M/$50M = 14% ***Return on Sales (Profit Margin) Net of Interest Expense*** =(EBIT – Interest Expense)/Revenue Company A: ($4M - $2M)/$20M = 10% Company B: ($7M - $5M)/$50M = 4% Profit margin is only an appropriate measure since the two firms in question are operating in the same industry. If the firms were not operating in the same industry, profit margin in itself would not be a valuable measure.	
Part b: 0.5 point	
• Sinking Fund – In order to spread the payment burden of the final principal payment over several years, the fund may be set up to repurchase a fraction of the outstanding bonds in the open market each year or at a special call price associated with the sinking fund provision. • Subordination of Further Debt – restricts the amount of additional borrowing. Additional debt is given lower priority to existing debt during a bankruptcy event. • Collateral – A particular asset can be specified that the bondholders will receive if the firm defaults on the bond.	

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT

For this question, candidates were expected to demonstrate knowledge on determinants of bond safety and provide basic financial ratios to support their thesis. Candidates were also expected to know what protective covenants an investor can use in the bond contract to protect themselves in the event of a default.

Part a

Candidates were expected to provide two of the financial ratios that identify the company that is less likely to default based on the results. Given the information provided, there were three possible ratios from which to chose: Return on assets, net of interest expense, Return on sales, Return on Sales net of interest expense.

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The most common error was to:

- Evaluate the ratio of Revenue/Assets. While this ratio can be useful to determine how a company is using its assets to produce sales, it is not useful in evaluating whether a company is likely to default since it ignores earnings.
- Providing Inverse of the Coverage Ratio without an adequate explanation of the candidate's thought process

Some candidates also tried producing debt-to-equity ratios or ratios involving balance sheet liabilities, but the given information in the problem was not sufficient to produce them.

Part b

Candidates were expected to briefly describe two protective covenants that can be added in the bond contract that would reduce loss to the investor when either company A or B default.

The most common error was listing dividend restrictions. While dividend restrictions can be a helpful tool to ensure a firm retains assets rather than paying them out to stockholders, the question specifically asked about reducing loss once a default occurs. Once a default occurs, a dividend restriction will not assist the investor in reducing loss.

Other somewhat common errors:

- Blocking future capital raises completely
- Incorrect description of Sinking Funds
- Credit Default Swaps
- Convertible bonds

While the sinking fund benefit is not as obvious, the presence of one would increase the likelihood that the bondholder had some of the original bonds redeemed prior to the default. When the default occurs, the investor would have a smaller amount of exposure to the loss.

QUESTION 11

TOTAL POINT VALUE: 2 | **LEARNING OBJECTIVE(S): C5**

SAMPLE ANSWERS

Part a: 1.5 points

Sample Responses for the identification and description of the triggers

- Indemnity Trigger – Bond payouts when losses incurred by insurer reach specified threshold.
- Index Trigger (Industry) – bond pays out when industry losses from the CAT reach a specified threshold.
- Index Trigger (Modeled Loss) – based on running a widely accepted CAT model over the insured locations.
- Index Trigger (Parametric) – Trigger based on a pre-defined event, like wind speed in hurricane.
- Index Trigger – Payments are triggers by loss experience of industry, loss model of the insurer or industry, or the parameters of the event.

Sample Responses for the Insurer Preference

- For the insurer, it is better to have the indemnity trigger since their losses has a low correlation with market so preferable to be based on actual losses (lower basis risk).

Sample Responses for the Investor Preference

- For the investor, it is better to take the industry index trigger since it has lower moral risk Index trigger is more transparent for the investor.

Part b: 0.5 point

Sample Reponses for advantages of Industry Loss Warranty (ILW) vs Non-Indemnity Bonds

- ILW is more likely to be treated like reinsurance for regulatory purposes.
- ILW are cheaper than CAT bonds (no need to setup SPV).
- Since they are not very correlated to the market, the straight non-industry triggers. expose them to too much basis risk. An ILW has dual triggers so they have presumably. arranged a lower industry threshold to protect themselves.
- ILW can be used to plug gaps in reinsurance programs.
- The ILW would pay off in the situation where both issuer's losses are high and the industry's losses are high and capacity may be limited.

EXAMINER'S REPORT

The candidates were expected to demonstrate understanding of the potential triggers for Catastrophe Bonds and what the advantage of these were from the insurer and investor

perspectives. Candidates were also expected to discuss advantages of an Industry Loss Warranty over a non-indemnity cat bond.

A common mistake included identifying only one of the advantages or preferences, or confusing the issuer and investor perspective.

Part a

Candidates were expected to identify two of the potential triggers for catastrophe bonds, describe them briefly, and then state which of the two would be preferable to the issuer and which would be preferable to the investor.

- .

Common errors included:

- Not describing the triggers
- Not providing both the insurer and investor perspective
- Reversing the insurer and investor perspective
- Stating 'index trigger' without a description of the index used
- Stating 'indemnity trigger' without a description that it was based on the actual loss of the insurer.
-

Part b

The candidates were expected to provide two advantages of an Industry Loss Warranty vs a non-indemnity cat bond.

Common mistakes included:

- Describing the ILW without discussing any advantages

  Example: "ILW has dual triggers that indemnify the insurer after a certain threshold"

- Giving advantages of the ILW that were not against the non-indemnity cat bond (i.e., ability to profit)

  Example: "ILW with a binary trigger may over indemnify insurer once both Industry & Warranty triggers are met, so there's an advantage for insurer to possibly get more than they lose."

- Giving advantages of the ILW from the perspective of the investor (i.e., lower moral hazard)

Example: "ILW has a binary trigger which may lower insurer threshold for moral risk"

- Only giving a single advantage, generally basis risk

EXAM 9 SPRING 2018 SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 12	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): C3 & C4**

SAMPLE ANSWERS

Part a: 0.75 point

Probability of default of a junior tranche (CDO)
= (0.15)^2 + 2 x 0.15 x 0.85
= 27.75%

Or
= 1- (0.85)^2
= 27.75%

Probability of default of a senior tranche (CDO2)
= 0.2775^2
= 7.70%

Price of investment
= [100 x (1-0.077)] / 1.05
= $87.90

Part b: 0.5 point

Probability of default of a senior tranche if perfect correlation
= 15%

Price of investment = [100 x (1-0.15)] / 1.05
= $80.95

Part c: 0.75 point

The CDOs should have higher expected return than single-name securities. The CDOs have higher exposure to systematic risk.

EXAMINER'S REPORT

Candidates were expected to calculate the price of a CDO, understand the impact of correlation on the pricing assumptions and, understand how CDOs' expected return is impacted by the embedded risk.

Part a

Candidates were expected to calculate the probability of default of a junior tranche and the probability of default of a senior tranche assuming uncorrelated securities. Then, candidates were expected to calculate the price on an investment in the uncorrelated senior tranche.

Common mistakes included:

- Not considering the probability of non-default in year 1
- Calculating price as probability of default x $100
- Not discounting the price of the investment

Part b

Candidates were expected to give the probability of default of a senior tranche given perfect correlation. Then, candidates were expected to calculate the price on an investment in the senior tranche also with perfect correlation.

Common mistakes included:

- Not assuming perfect correlation between securities
- Calculating price as probability of default x $100
- Not discounting the price of the investment

Part c

Candidates were expected to understand the impact of the CDO structure on expected return. Also, candidates were expected to describe why CDOs should have a higher expected return vs. single-name securities with the same rating.

Common mistakes included:

- Assuming lower expected return for CDOs
- Describing the impact on rating as opposed to expected returns
- Incorrectly justifying rationale for higher expected returns
    - Due to correlation (i.e. not describing the correlation to the overall economy)
    - Sensitivity to estimation errors
    - Error in the estimation of default or rating

QUESTION 13

TOTAL POINT VALUE: 1.25 | **LEARNING OBJECTIVE(S): C6, C7**

SAMPLE ANSWERS

Part a: 0.75 point

All calculations in millions

Risk Element	Amount	Capital Ratio	RBC
Stocks	$ 200	0.20	40
Bonds	$ 1,000	0.05	50
Total Assets	$ 1,200		

Risk Element	Amount	Capital Ratio	RBC
Loss Reserves	$ 800	0.35	280
Unearned Premium Reserves	$ 100	0.20	20
Total Liabilities	$ 900		

$$Diversified\ Capital = (40^2 + 50^2 + 280^2 + 20^2 + 2 * (40 * 50 * 0.2 - 50 * 280 * 0.4))^{0.5}$$
$$= \$\ 269.26$$

Part b: 0.5 point

Sample Answer 1:
Advantage: Contemplates severity of losses exceeding assets.
Disadvantage: Does not reflect diversification benefits.

Sample Answer 2:
Advantage: It's consistent with financial theory of pricing risky debt contracts.
Disadvantage: May not have enough capital to allocate to lines of business.

Sample Answer 3:
Advantage: Each line can be set to same EPD ratio.
Disadvantage: Need to select an arbitrary threshold.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of Risk Based Capital calculation as well as demonstrating understanding of the concept of capital allocation.

Part a

Candidates were expected to calculate the diversified risk-based capital taking into account the correlation of risk elements.

Common mistakes included:

- Incorrect risk-based capital formula
- Not accounting for the correlation
- Not realizing the correlation for Bonds – Loss Reserves results in a negative correlation

Part b

Candidates were expected to state an advantage and disadvantage of using EPD to allocate capital.

Common mistakes included:

- Stating that allocating capital using EPD was "intuitive" as an advantage.
- Stating that using EPD "was difficult because the calculations involved" or "was complicated to explain to management" as a disadvantage.

QUESTION 14

TOTAL POINT VALUE: 2.5 | **LEARNING OBJECTIVE(S): C7, C8, C9**

SAMPLE ANSWERS

Part a: 1 point

Sample 1 – Target RAROC at T=0

Adjusted RAROC = $R \sum C (1+y)^{-1} / C_1$

$= 0.16 * [6.5 / 1.04 + 6.5 * (.5 + .3) / (1.04)^2 + 6.5 * (.3) / (1.04)^3] / 6.5$

$= .3149$

Sample 2 – Target RAROC at T=0

Year	Capital Held	PV Factor	PV(Capital)
1	6.5M	$1.04^{-1} = .9615$	6.25
2	6.5(.8) = 5.2M	.9246	4.808
3	6.5(.3) = 1.95M	.89	1.734
			12.792

Target RAROC = .16 (12.792 / 6.5) = 31.5%

Sample 3 – Target RAROC at T=1

New RAROC = $0.16 * [(6.5 + 5.2 / 1.04 + 1.95 / 1.04^2) / 6.5]$

$= 32.75\%$

Sample 4 – Target RAROC at T=1

t	% Paid	Capital	Target RAROC	$ Target	D	Discounted Target
0	0	6.5	16%	6.5 (0.16) = 1.04M	1	1.04M
1	20%	5.2	16%	0.832M	1/1.04	0.8M
2	50%	1.95	16%	0.312M	$1/1.04^2$	0.288M
3	30%	0	16%	0		

Sum of Target = 1.04M + 0.8M + 0.288M = 2.128M

Multi year RAROC = 2.128M / 6.5M = 0.327 = 32.7%

Part b: 1 point

Sample 1 – Expected RAROC at T=0, Target RAROC at T=0

NI @ t = 0 -> $[25(0.85) - 4.2/1.04 - 10.5/1.04^2 - 6.3/1.04^3]/ 6.5$

$= 0.293$

$0.293 < 0.315$ = Not write the business

Sample 2 – Expected RAROC at T=0, Target RAROC at T=1

[25(1 - .15) - 21(.2/1.04 + .5/1.04^2 + .3/1.04^3)] / 6.5 = .293
.293 < .327 -> No, should not write the line of business.

Sample 3 – Expected RAROC at T=1, Target RAROC at T=1

PV of Loss @ Year 1 = .2(21) + .5(21)/1.04 + .3(21)/1.04^2 = 4.2 + 10.1 + 5.825 = 20.125M

[25*(.85)(1.04) - 20.125]/ 6.5 = 30.4%

RAROC < 32.7% Required Rate; should not write on this basis.

Sample 4 – Expected RAROC at T=1, Target RAROC at T=0

Economic Profit at t=1 = 25(1-15%)*1.04 - 21(0.2 + 0.5*1.04^{-1} + 0.3*1.04^{-2}) = 1.979M
RAROC = 1.979/6.5 = 30.4% < 31.5%
It's less than the target RAROC, so insurer should not write it.

Sample 5 – Expected Econ. Profit at T=1, Target Econ. Profit at T=0

Target Economic profit = 6.5M(.3149) = 2.0466M
Current Economic Profit (t=1)
Assume expenses pd immediately
[25(1-.15)] * 1.04 - 21(.2 + .5/1.04 + .3/1.04^2) = 1.979M

1.98M < 2.05M, so the insurer should not write this line of business

Sample 6 - Expected Econ. Profit at T=1, Target Econ. Profit at T=1 (Assume given discounted losses to T=1)

0.327 * 6.5 = 2.128 -> target profit
Insurers Economic Profit -> P - E - Dis Loss + (P - E) inv yield
(.85)(25) - 21 + (.85)(25)(.04) = 1.1 < Target economic profit; shouldn't write the line of business

Sample 7 - Expected RAROC at T=1, Target RAROC at T=0 (Assume given discounted losses to T=0)

Economic Profit = 25,000,000(0.85)(1.04) - 21,000,000 = 1,100,000
RAROC = Profit/Capital = 16.92%
Since it is lower than 31.49%, do not write the line.

Sample 8 - Expected RAROC at T=0, Target RAROC at T=0 (Assume given discounted losses to T=1)

Economic Profit = 25 (1-.15) - (21/1.04) = 1.058
RAROC = 1.058/6.5 = 16.27%

Since the RAROC of 16.27% < 31.5%, the multi-year adjusted RAROC, the insurer should not write this business, as it is not profitable.

Part c: 0.5 point

Sample 1

- One approach: ignore difference, use lifetime of insurance obligations & annual for market/asset risks
- Another: look only at one year horizon & view insurance risk as the change in liabilities (eg, incurred loss/reserve) over that period

Sample 2

- Can look at all risks (including market risk) over long term (potential lifetime of all items)
- Can look at all risks (including insurance liability risk) over 1 year period

Sample 3

- Can target all components with 1 year horizon
- Instead of aggregating, can take a "haircut" for market and credit risk

Sample 4

- Use capital modeling for multi-period horizons
- Binomial Trees – similar to Butsic's Expected Policyholder Deficit

Sample 5

- Dynamic financial analysis: simulate loss trajectory across multiple years and evaluate solvency and capital adequacy in the multiperiod analysis
- Rating Agency Model: practical measure like BCAR

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge on multi-period capital commitments and how that impacts the performance of a line of business. Additionally, candidates needed to demonstrate knowledge on how to combine risks with different exposure horizons within a risk capital model.

Part a

Candidates were expected to calculate the multi-year adjusted target RAROC.

Common errors included:

- Calculating the company's actual RAROC rather than the multi-year target
- Using the amount of capital released rather than the total capital remaining at the beginning of each year
- Using the incorrect discount rate

Part b

Candidates were expected to use a risk adjusted approach and, using their result from part a., make a recommendation on whether or not the company should write the business.

Common errors included:

- Timing mismatches within the calculation (i.e., discounting losses to time 0 but including investment income on net premiums)
- Comparing the calculated RAROC or EVA to the single-year target of 16%
- Not providing a recommendation

Part c

Candidates needed to provide a reasonable approach to combining different sources of risk with different exposure horizons within a risk capital model.

Common errors included responses related to adjusting the target RAROC and various rates of return, or stating how capital should be committed (i.e., 'capital is committed when policy is written and released when policy expires'). Such responses did not address how to combine different sources of risk within a capital model.

QUESTION 15

TOTAL POINT VALUE: 2 | **LEARNING OBJECTIVE(S): C6**

SAMPLE ANSWERS

Sample 1

Event		Loss Amount	P(Loss)	Cumulative Probability
1	None	0M	(.99)*(.85) = .8415	0.8415
2	Wind	5M	(.15)*(.99) = .1485	0.9900
3	EQ	15M	(.01)*(.85) = .0085	0.9985
4	Wind + EQ	20M	(.01)*(.15) = .0015	1.0000

99.5% VaR: Select 15M as capital requirement

Assign Capital by Event:

Event	Layer 1 5M xs 0	Layer 2 10M xs 5M	Layer 3	Total
2	$\frac{0.1485}{1-0.8415} * 5 = 4.6845$	--	--	4.6845
3	.2681	$\frac{0.0085}{1-0.99} * 10 = 8.5$	--	8.7681
4	.0473	1.5	--	1.5473

Assign Event Capital to Peril:

Event	Wind	EQ	Capital	Wind Alloc	EQ Alloc
2	100%		4.6845	4.6845	--
3		100%	8.7681	--	8.7681
4	25%	75%	1.5473	.3868	1.1605
				5.0713	9.9286

Sample 2

	Loss	Prob
None	0M	.8415
Wind	5M	.1485
EQ	15M	.0085
Wind + EQ	20M	.0015

99.5% VaR: Select 15M as capital requirement

5M to Allocate

Wind = $\frac{0.1485+\left(\frac{5}{20}\right)(0.0015)}{0.1585} * 5M = 4.696$M

EQ = 5 – 4.696 = 0.304M

10M Allocate

Wind = $\frac{\left(\frac{5}{20}\right)(0.0015)}{0.01} * 10M = 0.375M$

EQ = 10M – 0.375M = 9.625M

Total

Wind = 4.696M + 0.375M = 5.071M

EQ = 0.304M + 9.625M = 9.929M

EXAMINER'S REPORT

Candidates were expected to calculate the amount of allocated capital to two different peril exposures using the percentile layer methodology.

Common mistakes included:

- Using the incorrect loss amount as the 99.5% VaR
- Using the incorrect peril weights to split the combined peril loss event

QUESTION 16

TOTAL POINT VALUE: 4.5 | **LEARNING OBJECTIVE(S): C7, C9**

SAMPLE ANSWERS

Part a: 3 points

Sample 1

VaR(98%) for the Company = 200

For A:

In proportion to 98% VaR: 200 * 0/200 = 0

In proportion to 98% co-TVaR = 200 * (700 + 400)/(1000+500) = 147

Percentile Layer/ Scenario	97	98	99	100
100 XS 0	25	25	25	25
100 XS 100	0	33	33	33
Total	25	58	58	58

Scenario	A	B
97	0	25
98	0	58
99	46.7	11.7
100	40.8	17.5
	87.5 to Line A	

Sample 2

Reorder line B

300 100
200 99
100 98
100 97

VaR Allocation

98% Var = 0 – Line A → 0
200 – Line B → 200
200 – Aggregate

98% CoTVaR A = 1100/2 → 550
B = 400/2 → 200
Aggregate = 1500/2 → 750

^^Assume company changes its threshold to hold 98% aggregate TVaR as capital is not given

Percentile Layer – Assume company holds 200M in capital

100 x 0 for A = 100 * (.25*0 + .25*0 +.25*4/5 + .25 *7/10) = 37.5
100 x 100 for A = 100 * (.333 * 0 + .333*4/5 + .333*7/10) = 50.0 TOTAL = 87.5

Sample 3

98% Var = observation 98 = 200 total capital
VaR allocates 0 to line A (98% VaR_A = 0)

CoTVaR = avg top 3 (Bodoff includes the percentile threshold in his coTVaR)

Line	CoTVaR	% Alloc	X 200	Capital
A	366.67	.6471	X 200	129.41
B	200	.3529	X 200	70.59
	566.67			200

By Layer:

0-100		100-200	
Scenario	Alloc	Scenario	Alloc
97	25		
98	25	98	33.33
99	25	99	33.33
100	25	100	33.33

Capital Allocation

Scenario	Alloc	Line A	Line B
97	25		25
98	58.33		58.33
99	58.33	46.66	11.67
100	58.33	40.83	17.50
	200	**87.49**	112.50

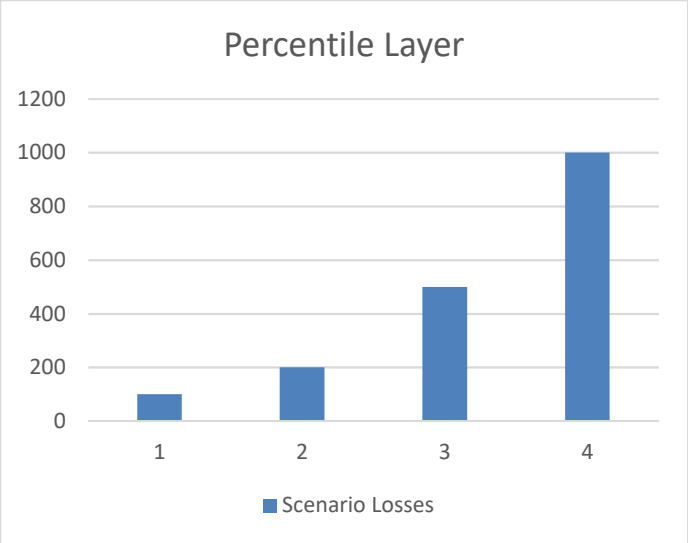
Sample 4
Capital at 98% VaR = 200

Proportional to 98% VaR
Line A gets 0

98% CoTVaR
A: (700+400)/2 = 550
B: (300+100)/2 = 200

Line A gets 550/750 * 200 = 146.67

Percentile Layer



100 X 0 layer

1. 100/4 = 25
2. 100/4 = 25
3. 100/4 = 25
4. 100/4 = 25

100 X 100 Layer

1.
2. 100/3 = 33
3. 100/3 = 33
4. 100/3 = 33

Line	1	2	3	4	Total
A	0	0	.8* (25+53)	.7 * (25+53)	87.5

Part b: 0.75 point

Sample 1

VaR- just looks @ one point and ignores the rest
Co-TVaR ignores loss lower than the given threshold (98%)
%ile layer allocation includes all loss events & probabilities

Sample 2

VaR looks at loss at a given level, as A has no loss at that level, not allocated capital. Co-TVaR looks at severities above the VaR threshold. A has large losses above the VaR threshold so it gets a larger portion of capital. Percentile Layer allocates capital based on all loss scenarios so A receives less than when using just scenarios contemplated in Co-TVaR

Sample 3

VaR focuses on a selected percentile, distorted because A doesn't have losses @ VaR(98%)
Co-TVaR reflect the average severity exceeding VaR for the company in aggregate and A makes up a larger portion of tail events.
Percentile layer allocates capital to the entire range of losses, rather than just extreme events.

Sample 4

i) VaR is a single point in the distribution such that P(Loss > VaR(.98)) = a and so for A, 98%-ile has no loss and results in no allocation

ii) Co-TVaR values the average loss in the tail and so gives a lot of weight to the large loss of 700 in A

iii) Percentile Layer recognizes that all loss levels contribute to risk and so scenario 97 also receives weight

Sample 5

i) The proportional to VaR 98% only looks at the 98% VaR for A and separately for B. For A, this is 0 loss even though A still has tail events at percentiles > 98% so A gets no capital.
ii) 98% co-TVaR – co TVaR looks at the worst results when sorted on total, and takes the average of events above the given percentile. We see the worst event for A is very large compared to B so A is getting more capital here.
iii) The Percentile approach looks at all scenarios and gives capital to the more frequent events but also to severe. B has more frequent events but A's events are more severe so the allocate capital is split more evenly between A and B

Sample 6

-using VaR only allocates capital to a specific VaR and doesn't care about above (more severe) or below (more scenarios)
-using co-TVaR focus on the severity above the 98% VaR but not anything below the 98%
-using percentile method reflects the severity above the threshold and also the frequency below the threshold

Sample 7

- VaR is only concerned w/1 pt, where line A happened to have no losses, so received no capital
- Co-Tvar uses the severity in the tail, which A has higher severity so A gets a majority of capital
- Percentile Layer method uses the entire distribution and allocates based on a mix of frequency + Severity

Sample 8

VaR only cares about loss at 98^{th} percentile and doesn't care about the rest of the distribution
Co-TVaR cares about the losses above the 98^{th} only
Percentile layer cares about the whole distribution of losses

Sample 9

VaR uses only outcomes above 98^{th} percentile, but line A only contributes to the 98^{th} so it gets 0 capital
CoTVaR uses only outcomes above the 98^{th} percentile to which A contributes disproportionately, so A gets more capital.

Percentile layer uses all outcomes so in general is in between, depending on how much A and B contribute to the layers

Part c: 0.75 point

Sample 1

P = E(L) + r(Allocated Capital – Contributed Capital)
= (700+400)/100 + .15/1.15(87.47- ((700+400)/100)
= 20.98

Sample 2

E[L] = (700+400)/100 = 11

P – Expense = 11 + .15(87.5- (P-E))
x = 11 + 13.125 -.15x
1.15x = 24.125
X = 20.978

Sample 3

Expected Losses = (400 + 700)/100 = 11
Assume no investment return, (P-E) for Line A = 87.5 x 15% + 11 = 24.125

Sample 4

P = Premium net of expense
P = E(L) + r(Allocated capital – P) E(L) = (400 + 700)/100 = 11
P = 11 + 15%(87.47-P) → P = 20.97

Sample 5

Expected Loss: (700+400)/100 = 11

Cost of Capital: 15%

P = 11 + 15%(87.5 – P)
P = 20.98

Sample 6

87.5 Capital → (P – 11)/87.5 = .15 P = 24.125 ignoring investment income

Sample 7

P = E(A) + .15/1.15 x (87.5 – E(A))

E(A) = 1100/100 = 11

P_A = 20.98

EXAMINER'S REPORT

Candidates were expected to understand the measures of capital allocation asked for in the problem and apply those measures to allocate a selected risk capital at a specific threshold. Candidates were then expected to evaluate characteristics of the different methods that led to the different results. Finally, candidates were expected to apply the concept of RAROC to calculate the premium that should be charged.

Part a

Candidates were expected to select the 98th simulation of Aggregate UW Risk as the 98%ile VaR threshold and then correctly allocate a dollar amount to Line A.

Candidates were expected to calculate the proportion of VaR and Co-TVaR belonging to Line A and then apply that to the aggregate capital level. Candidates should differentiate between these two risk measures. For Co-TVaR, both Bodoff's approach of including the simulation at the VaR and Goldfarb's approach of using only the simulations further in the tail were accepted. For the percentile Layer method, candidates were expected to apply the method properly to allocate the 98th percentile VaR.

Common mistakes included

- Selecting the wrong simulation for the VaR
- Allocating using T-VaR measure proportionally rather than the Co-TVaR
- Allocating the wrong capital amount for Co-TVaR
- Calculating the Percentile Allocation using layers beyond the selected threshold
- Not understanding the Percentile layer allocation
- Assuming the simulation table was provided for 3 different lines rather than two lines and for the total of the two

Part b

Candidates were expected to evaluate the results of part a. and connect the different outcomes to common traits of the VaR, Co-TVaR and Percentile Layer allocation methods that gave rise to the results.

Common mistakes include:

- Ignoring the specifics of the calculated answer for part a.
- Defining a measure, such as "The VaR selects a point that has a certain exceedance probability" or "The Co-TVaR is the average loss greater than the VaR threshold"
- Listing a characteristic that did not distinguish the different results in part a. For example, stating that Co-TVaR takes into consideration losses in the tail. However, the Percentile Layer method also considers loss events in the tail.

Part c

Candidates were expected to derive an expected loss from the table of simulations and then calculate a premium using the method in Bodoff that would yield the required return on allocated capital from the Percentile Layer method in part a.

Common mistakes include:

- Trying to apply a RAROC formula without an investment income assumption
- Incorrectly calculating the expected loss
- Using capital from the wrong line of business or risk measure from part a. above
- Using the total E(L) from above
- Calculation errors in the formula

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QUESTION 17	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE(S): C4 & C5**

SAMPLE ANSWERS

Part a: 2.75 points

Let's calculate surplus for period 1 and period 2 =>
PV Unpaid Losses;
36/1.08 + 36/1.08^2 = 64.20
Surplus = 0.30*64.20 = 19.26

For year 2; unpaid losses = 36/1.08 = 33.33
Surplus = 10

We need GAAP equity: assuming DAC = 18
GAAP equity =>
Time 0 => 18 + 19.26 = 37.26
Time => Q1 = S1 = 10

Now we need net income:
U/W GAAP Income:
Time 1: EP – IX – IL
100 – 72 – 30 = -2

Time 2: 0 – 0 – 0 = 0

Investment Income =>
Time 1: 120(0.06) = 7.2
Time 2: 75(0.06) = 4.5

Total Income after Tax:
Time 1: (-2 + 7.2)(1-0.35) = 3.38
Time 2: (4.5)(1-0.35) = 2.925

PVI/PVE = (3.38 + 2.925/1.10) / (37.26 + 10/1.1) = 6.0391 / 46.351 = 0.1303
13.03%

Part b: 0.25 point

Possible answers include:

- PVI/PVE = IRR when $r_I = r_Q$ = IRR. i.e., discount rate for income equals discount rate for equity and also equals internal rate of return.
- If the PVI/PVE discount rate = IRR.
- This will occur when the PVI/PVE discount rate, which equals the cost of capital, is equal to the IRR.
- PVI/PVE will equal IRR if the target PVI/PVE, PVI/PVE discount rate, and target IRR are all set to the cost of capital.

EXAMINER'S REPORT

Candidates were expected to calculate the PVI/PVE and recognize when its results are equivalent to the IRR method.

Part a

Candidates were expected to calculate PVI/PVE by deriving the appropriate discounted income and equity with the information provided. This required recognizing GAAP incurred expenses at time 1, including DAC in Equity and discounting to the correct time periods with the appropriate rates provided.

Common mistakes include:

- Not recognizing UW Expenses are all incurred at time 1 for use in the GAAP UW Income calculation.
- Using paid losses rather than incurred losses to compute GAAP UW Income.
- Recognizing Investment Income at time 0 and 1 rather than time 1 and 2.
- Not recognizing the appropriate DAC for GAAP equity.
- Incorrectly discounting unpaid losses or not discounting unpaid losses for calculating surplus.
- Discounting income and/or equity to the wrong time period or using the wrong discount rate.
- Computing investment income on Assets derived with insufficient information rather than the Investible Assets given.

Part b

Candidates were expected to understand the relationship between PVI/PVE and IRR.

Common mistakes included stating:

- "Cost of Capital = IRR" without clarifying that the cost of capital is used to discount both income and equity.
- PVI or PVE discount rate = IRR; these discount rates can be different by definition.
- Investment Yield = IRR
- Reinvestment Rate = IRR
- Growth Rate = IRR
- This occurs when insurer is in a steady state
- UW Profit set so that IRR is achieved; this does not describe the circumstances when that is true.

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QUESTION 18

TOTAL POINT VALUE: 1.25 | **LEARNING OBJECTIVE(S): D3**

SAMPLE ANSWERS

Part a: 0.75 point

	Premium Proposed	Expected Loss and Expense	Premium to Surplus Ratio	Profit	Surplus	ROE
Company A	105	94	2:1	11	52.5	21.0%
Company B	105	94	1:1	11	105	10.5%
Company C	98	94	3:1	4	32.7	12.2%

Company A is rejected.

Part b: 0.5 point

Two of the following:

- ROE method requires artificial allocation of surplus
- ROS method is analogous to markup, which is intuitive to consumers
- ROE method puts more focus on rate-of-return equity, instead of rate equity
- ROE method is just a convoluted way of regulating ROS
- ROE regulation will mean the companies with high surplus will be able to charge higher rates
- The ROE method could produce different results for identical filings because of varying capital structures
- ROE punishes companies with more capital as they will have to charge higher premiums to be considered rate adequate

EXAMINER'S REPORT

Candidates were expected to use the return-on-equity method to assess insurance profitability and understand the disadvantages of the return-on-equity method in comparison to the return-on-sales method.

Part a

Candidates were expected to calculate the return-on-surplus for each company and conclude that Company A is above the threshold and will be rejected by the regulator.

Common errors include:

1. Using an incorrect formula to calculate the return-on-surplus (for example, using premium instead of surplus in the denominator)
2. Concluding that companies B & C would be rejected by the regulator

Part b

Candidates were expected to describe two disadvantages of the return-on-equity method compared to the return-on-sales method.

Common errors include:

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1. Concluding that the return-on-equity method would impose lower rates on companies with higher surplus
2. Confusing rate adequacy with rate equity

QUESTION 19	
TOTAL POINT VALUE: 2.25	**LEARNING OBJECTIVE(S): D6**

SAMPLE ANSWERS

Part a: 1.75 points

Sample

Surplus allocated to line B according to 1% TVaR:
(3,435,305 – 1,132,936)/4,562,868 = 50.5% allocated to line B net of reinsurance.

Allocated Surplus = $8M * 50.5% = $4.036M

Mean return = $375k - $125k = $250k

ROS = $250k / $4.036M = 6.19%

Part b: 0.5 point

Sample

Currently holds: $8M
New surplus requirement = 1.2*$4,562,868 = $5,475,442
Can release: $8M - $5,475,442 = $2,524,558

EXAMINER'S REPORT

Candidates were expected to understand the relationship between capital needs and risk loads, and evaluate a reinsurance purchase from the cedant perspective.

The question required candidates to make correct assumptions about the surplus basis and risk capital metric.

Part a

Candidates were expected to understand the relationship between capital needs and risk loads, and evaluate a reinsurance purchase from the cedant perspective. Candidates needed to correctly assume that current surplus was proportionally allocated to B net of reinsurance based on 1% TVaR.

Common mistakes included:

- Assuming target surplus instead of allocating current surplus.
- Ignoring the cost of reinsurance in mean return, or the benefit of reinsurance to allocated surplus.
- Averaging TVaR levels, allocating investment income to B's mean return and using the wrong % line for their TVaR calculation

Part b

Candidates were expected to reduce current surplus to the target surplus based on the total 1% TVaR. Averaging of VaR's in the first column at or below the 1% level to calculate the TVaR was unnecessary but allowed.

Common mistakes included:

- Calculating the target surplus of reinsurance.
- Averaging TVaR's

QUESTION 20

TOTAL POINT VALUE: 1.75 | **LEARNING OBJECTIVE(S): D7**

SAMPLE ANSWERS

Part a: 0.5 point

Sample 1

R_b = y(var(b) + covshare(B)) = 1.07

= y(7.19 + 8.09) = 1.07

y= 0.07

Sample 2

2.024 = m(16.76+12.15)

m = 0.07

Sample 3

Method is renewal additive

Total risk load = 2.024 + 1.07 = 3.094

Total variance = 16.76 + 7.19 + 12.15 + 8.09 = 44.19

X = 3.094/44.19 = 0.070016

Part b: 1.25 points

Sample 1

Event	P	A	B	CS(A)	CS(B)
1	0.005	15	8	0.78	0.41
2	0.01	15	10	1.78	1.19
3	0.01	15	15	2.23	2.23
4	0.015	15	7	2.12	0.99
5	0.02	12	7	2.08	1.21
6	0.025	8	9	1.65	1.86
total				10.64	7.89
variance		13.28	7.19		

risk load for x = 0.07*(13.28+10.64) = 1.6744

risk load for y = 0.07*(7.19+7.89) = 1.0556

overall change = 2.73 - 3.094 = -0.364 million

Sample 2

Event	P	A	B	Var A	covar(A,B)
1	0.005	15	8	1.119	0.597
2	0.01	15	10	2.228	1.485

3	0.01	15	15	2.228	2.228
4	0.015	15	7	3.324	1.551
5	0.02	12	7	2.822	1.646
6	0.025	8	9	1.56	1.755
Var=			7.19	13.281	9.262

1.119=15^2*0.005*0.995
0.597=15*8*0.005*0.995
Var(B) remains unchanged

2*cov(a,b) = 18.524

total risk load = 0.07003*(13.28+7.19+18.524)
new = 2.73M
old = 3.092M = 2.024+1.07

Reduction in RL = 0.364M

Sample 3

Risk load = multiplier * var(A+B)
Delta risk load = multiplier * delta in var(A+B)

Current var(A+B) = risk load/multiplier
= (2.024+1.07)/0.07 = 44.2

Event	A	B	A+B	prob
1	15	8	23	0.005
2	15	10	25	0.01
3	15	15	30	0.01
4	15	7	22	0.015
5	12	7	19	0.02
6	8	9	17	0.025

Var(A+B) = sum($L_i^2 - r_{i*}*(1-r_i)$) = 39
Delta in var(A+B) = 44.2-39 = 5.2
Delta in risk load = 5.2*0.07 = 0.364M

Risk load will decrease by 0.364M

EXAMINER'S REPORT

Candidates were expected to know how to use the covariance share method to calculate risk loads and to understand how to determine a risk load multiplier given relevant information.

Part a

Candidates were expected to calculate the risk load multiplier, using the variance and covariance share of A or B.

Common mistakes include

- Using the wrong risk load formula
- Trying to calculate the risk load multiplier from the marginal surplus method

Part b

Candidates were expected to restate the loss distribution for channel A according to the reinsurance contract described. Candidates were expected to compute the components of the risk load formula according to the new distribution to get the overall risk load and calculate the change from the original risk load.

Common mistakes include

- Calculating the change in risk loads for channel A only
- Not using the binomial approximation to calculate var(A) or var(A+B)
- Not restating the variance of A
- Not stating the sign of the risk load change (negative sign or reduction)
- Forgetting to calculate the overall change in risk loads (only calculating the overall risk load)
- Using the wrong formula to calculate the covariance shares

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QUESTION 21

TOTAL POINT VALUE: 1.25 | **LEARNING OBJECTIVE(S): D2**

SAMPLE ANSWERS

Part a: 0.75 point

Sample 1

T/S (Return on Equity) = Investor's viewpoint. Investors try to maximize return on equity for their investment.
I/A (Return on Assets) = Society's viewpoint. Society uses this to examine if the company has used its capital efficiently.
U/P (Return on Sales) = Regulator/Actuary viewpoint. Regulator wants to make sure the rate is not excessive or inadequate. Actuary wants to make sure it is adequate.

Sample 2

T/S -> ROE = Shareholder's viewpoint. Interested in the return on the surplus that they provide.
I/A –> ROA = Economists viewpoint. Interested in how the company's assets are being used.
U/P –> ROS = Management's viewpoint. Interested in maximizing company's profit margin.

Part b: 0.5 point

- An increase in P/S will impact I/A. I/A will decrease because 1) more assets are stuck in agent's balances and non-investable assets and 2) increased leverage leads to more insurance risk which should be countered by a more conservative investment strategy to reduce overall risk of the firm.
- When U/P increases, I/A will likely increase too because strong underwriting returns allows the company to adopt a more aggressive investment strategy.
- As U/P increases, P/S tends to increase as well. When profit margins are high, company is interested in writing additional business.
- When P/S increases, U/P may decrease. Company growth may be the result of loosened underwriting standards.

EXAMINER'S REPORT

Candidates were expected to demonstrate their understanding of the various returns and ratios in the provided equation, how they are used and how they interact with each other.

Part a

Candidates were expected to correctly identify all 3 returns in the provided equation, the viewpoint for each return as well as a brief explanation as to why each return metric was important to the shareholder, the economist or management..

Common mistakes included:

- Not providing an explanation along with the viewpoint for the return
- Stating that I/A was the viewpoint of investors/shareholders
- Identifying P/S as a return
- Stating that U/P was the viewpoint of the insurance company (too vague)

Part b

Candidates were expected to list two interactions between a ratio that involved premium (P) and another in the equation in order to conclude that P changes have an effect on total return.

Common mistakes included:

- Lack of details in the explanation of the interactions.
- Not explaining why a company adopts a more conservative investment strategy when P/S increases (more risk)
- Providing the same interaction twice with two different explanations (P/S --> I/A)
- Explaining impacts of ratios on the value of the firm rather than the ROE