Dependence Models and the Portfolio Effect

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Abstract

This paper describes efforts to estimate the "portfolio effect" — the diversification benefit from assembling a portfolio – by simulating the implied portfolio-level capital safety standard for various contract-level capital safety standards. The results showed that apparently aggressive contract-level capital standards still implied conservative portfolio-level capital safety standards. Taken at face value, this would have had a dramatic impact on pricing decisions.

However, the method used to generate the simulated contract outcomes — the Normal copula — was found to generate asymptotically independent tail samples, thus understating the tail of the portfolio outcome distribution. Tail-based risk measures were, therefore, understated as well.

This provides compelling evidence why actuaries must utilize alternative dependence models beyond the Normal copula.

Key words: dependence models, Normal copula, portfolio effect, capital allocation.

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1. Introduction

Many re-insurers assess the price of their business using some form of contract-level capital allocation — e.g., ruin threshold, marginal standard deviation, expected policyholder deficit ("EPD") [1], or tail conditional expectation ("TCE") [4]. Practical application of any of these capital measures in contract pricing requires (i) stochastically modeled contract outcome distributions, and (ii) a selected "safety standard" for that risk measure (e.g., 99^{th} percentile for ruin threshold). The more stringent the safety standard, the more capital will be allocated. But what contract safety standard should a company use? And what *portfolio* safety standard does that contract-level standard imply? In other words, what is the "**portfolio effect**" — the diversification benefit of writing a contract as a part of a large portfolio rather than on its own?

This paper presents results of a simulation study of the relationship between contract-level safety standards and the implied portfolio safety standard for the expected *policyholder deficit and ruin threshold risk measures. The study uses a simulation model* of a portfolio of reinsurance contracts programmed in the S-Plus language[1].

Using a standard technique for generating a multivariate log-Normal sample with correlation — the "**Normal copula**" — across a reasonable range of inputs, apparently *aggressive contract-level safety standards roll up to prudent portfolio-level safety* standards. Similarly, more conservative contract-level safety standards roll up to extremely conservative portfolio safety standards.

Taken at face value, these results challenge popular thinking about reinsurance pricing using capital allocation, as the portfolio effect is greater than anticipated. However, upon deeper analysis, it appears the effect may be overstated due to limitations in the dependence modeling implicit in the Normal copula. In other words, an accepted "standard" actuarial simulation technique understated the tail of the portfolio outcome distribution. Tail-based risk measures were, therefore, understated as well.

This provides compelling evidence why actuaries must utilize alternative dependence models beyond the Normal copula.

This paper proceeds through six additional sections. Section 2 presents an overview of the study. Section 3 provides details of the study, and Section 4 describes in depth all the calculations for a single example iteration of the study. Section 5 explains the results

[1] The results of the study in Excel pivot tables, as well as the S-Plus script file (program), will be posted on the CAS website.

of the study. Section 6 addresses the concept of dependence modeling. Section 7 gives conclusions.

2. Overview of the Study

This study was prompted by efforts at American Re to calculate risk-based capital amounts for individual contracts. The methods tested were expected policyholder deficit ("EPD") and ruin threshold. To calculate contract capital using a particular method, a contract-level safety standard is needed. An EPD standard based in part on A.M. Best information was available at the portfolio level, and there were several popular anecdotal ruin thresholds (e.g., 99^{th} percentile).

Clearly the portfolio-level standard would be too conservative to use at the contract level, due to diversification — the elusive portfolio effect. But how much should the standard be relaxed at the contract level? And is there any way to tie the selected contract standard to the portfolio standard? In order to make an informed decision, and to test the assumptions, we conducted the simulation study described below.

The results were surprising and contrary to our expectations. Because of the widespread use of similar techniques in re-insurance (and some primary insurance), we felt it would be beneficial to put the details and results of the study into the public domain. It is our hope that this study will prompt deeper discussion about choices of a dependence model with respect to diversification and capital allocation.

3. Details of the Study

The impact of these four variables was studied:

A. Individual contract expected loss
B. Aggregate portfolio standard deviation
C. Inter-contract correlation measure
D. Contract-level risk measure standard

For each iteration of the simulation, we selected a value for each of these variables.

A. Individual Contract Expected Loss
We modeled the portfolio as comprised of identical contracts of various expected loss amounts. We tested seven different individual contract expected loss amounts:

\$5M, \$10M, \$15M, \$25M, \$50M, \$75M, and \$100M

We assumed the entire portfolio of contracts had a total expected loss of **\$1 Billion**. Given that amount, the choice of an average contract size determines how many contracts of that size make up the portfolio. For example, there would be two hundred $5M contracts, one hundred $10M contracts, etc.

B. Aggregate Portfolio Standard Deviation
We tested aggregate portfolio coefficients of variation ("CV's") of **0.29, 0.32, and 0.36**. We considered these to be reasonable values for the overall portfolio variability. Given the $1B total expected loss, these CV's determined the portfolio standard deviation.

C. Inter-contract Correlation Measure
We tested three different inter-contract correlation levels for input to the multivariate Normal copula: **15%, 20%, and 25%**. Since the process involves generating Normal samples, then exponentiating these to derive log-Normal samples, these measures in fact represent the correlation between the log of the contract outcomes. We assumed this correlation was constant between all contracts.

D. Contract-level Risk Measure Standard
We tested the following levels for EPD and ruin threshold:

EPD: **20%, 15%, 10%, 7.5%, and 5%**
Ruin: **15%, 12.5%, 10%, 7.5%, and 5%**

Contract Loss Distributions
Given:

A. *Contract Expected Loss* (hence number of contracts),
B. *Aggregate Portfolio Standard Deviation*, and
C. *Inter-contract Correlation*,

individual contract variance is uniquely defined. Aggregate portfolio variance is the sum over the entire covariance matrix. The diagonal elements of the covariance matrix are the individual contract variance (assumed constant). Each off-diagonal element is the individual contract variance multiplied by the inter-contract correlation (assumed constant). Thus, for N contracts,

Contract Variance = v
Aggregate Portfolio Variance = V
Inter-contract correlation = ρ

$$V = Nv + \rho N(N-1)v = v[N + \rho N(N-1)]$$
$$v = V / [N + \rho N(N-1)]$$

We assumed a log-Normal distribution for the individual contracts, because it is a skewed distribution that represents aggregate contract loss distributions reasonably well. It is also straightforward to generate correlated log-Normal samples using the multivariate Normal distribution. We determined the μ and σ parameters for the log-Normal using moment matching.

Contract-level Capital
For each iteration, we selected a Total Asset amount ***A*** for each contract, based on either EPD or ruin threshold. For example, a ruin threshold of 99% (1% ruin probability) for a log-Normal distribution with known parameters is simply the 99^{th} percentile of that distribution. This amount would be ***A***.

A is composed of premium and capital. For purposes of the study, we assumed the premium amount was the individual contract expected loss amount, implying contract capital $C = A - E[L]$.

Implied Portfolio Capital
The implied portfolio capital is the sum of the calculated individual contract capital amounts. The portfolio expected loss is the sum of individual contract expected loss amounts. The sum of these two items gives the portfolio asset amount. In order to determine what risk measure standard this total asset amount corresponds to, we needed to determine an aggregate portfolio loss distribution. We did this using simulation.

Using the μ and σ parameters and the selected inter-contract correlation, we generated 5,000 samples from a multivariate Normal distribution with the number of variables equal to the implied number of contracts. Log-Normal samples were then created by exponentiating the generated Normal samples. For each iteration, the sum of these log-Normal sampled loss amounts is the simulated portfolio total loss.

The 5,000 iterations produce an empirical portfolio aggregate loss distribution. We could then calculate the risk measures using this distribution. Portfolio ruin probability is estimated as the number of iterations where portfolio loss exceeded the total portfolio assets divided by the total number of iterations. Portfolio EPD is the expected value of the amount by which portfolio loss exceeded the total assets.

4. Detailed Explanation of an Example Iteration

This section provides details of a single example iteration of the study. As stated above, for each set of simulations we selected a different scenario from each of four variables:

A. Contract Expected Loss (7 possibilities)
B. Aggregate portfolio standard deviation (3)
C. Inter-contract correlation (3)
D. Contract-level risk measure standard (5)

In the actual study, the simulation was repeated 315 times ($7 \times 3 \times 3 \times 5$). For this example, we will select one value from each of the above variables.

A. Contract Expected Loss
In this case, we will use **$10M** as our individual contract expected loss. Since we are keeping our aggregate expected loss fixed ($1B), this individual contract expected loss implies 100 contracts.

B. Aggregate portfolio standard deviation
We used different CV scenarios to come up with our implied aggregate portfolio standard deviation. Here we will use a **0.32** CV, which implies an overall portfolio standard deviation of $320M.

C. Inter-contract correlation
For this example we will use **0.20**. We already have the first moment of our individual contract loss distribution by assumption ($10M). Our selection of inter-contract correlation, combined with our assumption with respect to aggregate portfolio standard deviation, implies a unique second moment for our individual contract loss distribution.

Contract Variance = v
Aggregate Portfolio Variance = $V = (320M)^2$
Inter-contract correlation = $\rho = 0.20$
Number of Contracts = $N = 100$

$$V = Nv + \rho N(N-1)v = v[N + \rho N(N-1)]$$
$$(320M)^2 = v[(100) + (0.20)(100)(99)]$$
$$v = (320M)^2 / (2080)$$
$$v = (7.016M)^2$$

An intuitive way to visualize this is to picture our 100 × 100 covariance matrix, which represents our entire portfolio of contracts. By assumption, the sum of this matrix must add up to $(320M)^2$. In the case of independence, only the diagonal of our covariance matrix would be populated and our individual contract variance would simply be equal to V / N. As we increase the correlation, the variance along the diagonal becomes diluted as we spread more and more of the total variance to the off-diagonal cells in our matrix. The $\rho N(N-1)$ term in the above expression represents the strength of this dilution.

D. Contract Loss Distributions
Our individual contracts have an expected loss of $10M with a variance of $(7.016M)^2$. This implies a contract coefficient of variation of 0.7016. Since we are assuming a log-Normal distribution for individual contracts, we can solve for the σ parameter by using the following relationship.

$$CV = \sqrt{e^{\sigma^2} - 1}$$
$$\sigma = \sqrt{\ln(CV^2 + 1)} \qquad (1)$$

where CV is the coefficient of variation.

The σ parameter for our contracts is the square root of $\ln[(0.7016)^2 + 1]$ which is equal to 0.6327. Similarly, we can solve for the μ parameter using the following relationship.

$$E[L] = e^{\left(\mu + \frac{\sigma^2}{2}\right)}$$

$$\mu = \ln(E[L]) - \frac{\sigma^2}{2} \qquad (2)$$

where $E[L]$ is the expected loss.

For our example, we know the expected loss is \$10M and we know σ is equal to 0.6327. This implies

$$\mu = \ln(\$10M)-(0.5)(0.6327)^2 = 15.918.$$

Contract-level risk measure and safety standard

We'll examine the above individual contract at a 10% expected policyholder deficit. A similar exercise can be performed for probability of ruin. We will take this result for the individual contract, and multiply it by the number of contracts in our portfolio to get total implied capital for our portfolio.

The deficit (D) for a given contract with a certain amount of assets (A) allocated to it, and an uncertain loss amount (X) can be defined as:

$$D = \begin{cases} X - A & X > A \\ 0 & X \le A \end{cases}$$

If $f(x)$ is the density function for the loss variable X, then the expectation of the deficit, D, is:

$$\begin{aligned} E(D) &= \int_{-\infty}^{\infty} D \cdot f(x)dx \\ &= \int_{0}^{A} (0) f(x)dx + \int_{A}^{\infty} (x - A) f(x)dx \\ &= \int_{A}^{\infty} (x - A) f(x)dx \\ &= \int_{A}^{\infty} (x) f(x)dx - \int_{A}^{\infty} (A) f(x)dx \end{aligned}$$

$$= -\int_0^A (x)f(x)dx + \int_0^A (x)f(x)dx + \int_A^\infty (x)f(x)dx - A[1-F(A)]$$

$$= \left[\int_0^A (x)f(x)dx + \int_A^\infty (x)f(x)dx\right] - \left[\int_0^A (x)f(x)dx + A[1-F(A)]\right]$$

$$= E(X) - E(X \wedge A)$$

where ***E*(*X*^*A*)** is the expected value of ***X***, limited to ***A***.

Typically, the expected deficit is expressed as a percent of expected loss, ***E*(*X*)**. This gives us:

$$EPD\% = \frac{E(X) - E(X \wedge A)}{E(X)} = 1 - \frac{E(X \wedge A)}{E(X)} \qquad (4)$$

For our specific example we have an EPD percent of 10% and the log-Normal parameters of our individual contract loss distribution μ and σ are 15.918 and 0.6327, respectively. We need to solve for ***A***, individual contract assets. This can be determined either via simulation or through numerical methods. In our case, ***A*** is equal to approximately $16.2M.

<u>Implied Portfolio Capital and Safety standard</u>

Continuing our example, assuming we write 100 identical $10M contracts, the implied portfolio capital would be 100 × $16.2M or $1.62B. The final question is, "How 'safe' is the portfolio?"

To create the distribution of losses for our portfolio, we simulated from a multivariate Normal distribution using our individual contract parameters and the selected correlation ρ, in this case 20%. In this example, since we have 100 contracts, each iteration of the simulation produces a vector of length 100. This vector is exponentiated, then summed. This procedure is repeated 5000 times to produce the loss distribution for our portfolio.

Using this loss distribution for the portfolio, it is a simple exercise to solve for EPD% in the above expression using A = $1.62B. For our example, using the simulation results from the study, this was equal to 0.0067 or 0.67%.

5. Results of the Study

Pivot Table of Results

Given the number of dimensions in motion here (four), the best way to assess the results is with a pivot table. A Microsoft Excel 97 file with pivot tables of results for ruin threshold and EPD is posted on the CAS website (www.casact.org). The pivot table allows the user to select aggregate portfolio CV and inter-contract correlation values. The tables then display contract expected loss down the column, and contract-level risk measure across the row. The table itself shows the resulting portfolio risk measure.

Tables 1 - 6 show the EPD and ruin threshold results for selected aggregate portfolio CV's and inter-contract correlation values.

S-Plus Script

S-Plus is a statistical programming environment produced by Insightful Software (www.insightful.com). The S language was first developed by Bell Labs. S-Plus is used extensively in the statistical community. It is a vector-based language with substantial statistical and simulation capabilities. It handles large amounts of data well, and runs large-scale simulations quickly. The script file is also on the CAS website for others to use or modify and extend the analysis.

Implications

For a range of reasonable input assumptions, aggressive contract-level safety standards (e.g., 10% EPD) appear to roll up to prudent portfolio-level safety standards. Similarly, more conservative contract-level safety standards (e.g., 1% EPD) roll up to extremely conservative portfolio safety standards.

For example, if the company wished to hold capital commensurate with an A rating (roughly corresponding to an EPD of 0.5%), they could use the study results to support contract safety standards anywhere from 5.0% to 10.0%. Similar examples could be found using ruin threshold. The implications of implementing such contract safety standards in pricing are dramatic. These results were far from those expected by underwriters and pricing actuaries. They were also far from the standard in use at the time of the study (1% EPD). Implementing even the most conservative standard — 5% EPD — would have represented a dramatic shift.

Whenever indications deviate dramatically from current figures, both sets are called into question. The same phenomenon occurred here; the divergence of indications from current values led us to backtrack and analyze each component step in the simulation study. The range of input assumptions held up under further review. However, the seemingly innocuous choice of simulation method did not.

6. Dependence Modeling

As described previously, our approach to generating multivariate log-Normal samples relied upon the Normal copula and linear correlation. This approach qualifies as de facto "standard practice" for many simulation exercises carried out by North American actuaries. We are familiar with the multivariate Normal distribution and linear correlation from our exam syllabus, and software products to generate samples from this distribution are widely available (e.g., Microsoft Excel with @Risk, S-Plus). That makes it familiar and convenient, but is it any good? Does it produce appropriate results?

Risk and capital measures focus on the tails of distributions, so simulation techniques should reasonably model aggregation risk as reflected in the tail of the portfolio distribution. We relied on the Normal copula to model that risk. Our results were to a large extent a function of the mechanics of the Normal copula and its implicit *dependence model*.

The concept of *stochastic dependence measures* is not on the North American actuarial syllabus yet. Correlation is, but correlation is only one measure from this broader and more general class. Quoting Embrechts et al [2]:

> "Some of the confusion [surrounding correlation] may arise from the literary use of the word to cover any notion of dependence. To a mathematician correlation is only one particular measure of stochastic dependence among many. It is the canonical measure in the world of multivariate normal distributions, and more generally for spherical and elliptical distributions. However, empirical research in finance and insurance shows that the distributions of the real world are seldom in this class." [2, p. 2]

In other words, linear correlation completely describes the dependence relationship among the variables for the classes of elliptical and spherical distributions, of which the multivariate Normal is a member. However, most skewed distributions – including the log-Normal – are not members of these classes. So the dependence relationship between individual variables in a multivariate distribution from non-elliptical and non-spherical classes is *not fully described* by the linear correlation matrix.

Asymptotic Tail Independence in the Normal Copula

Of particular concern to actuaries performing simulation studies is the asymptotic tail independence of the Normal copula. Section 4.4 of Embrechts et al [2] discusses this at length. Summarizing their conclusions:

> "Thus the Gaussian [Normal] copula gives asymptotic independence, provided that $\rho < 1$. Regardless of how high a correlation we choose, if we go far enough into the tail, extreme events appear to occur independently in each margin." [2, p.19]

This is an alarming conclusion. Most actuarial risk measures focus on the tails. Any multivariate simulation exercise that systematically generates essentially independent tail samples will understate aggregate tail probabilities and, thus, understate the risk measure. The very portion of the curve we are focusing on is not being modeled properly by this "familiar and convenient" method.

A simple example will help reinforce this important concept. Figure 1 shows the plot of a 5000 point sample generated from a bivariate Normal distribution with $\mu = (12, 12)$, $\sigma = (0.5, 0.5)$, and correlation = 70%.

Figure 1

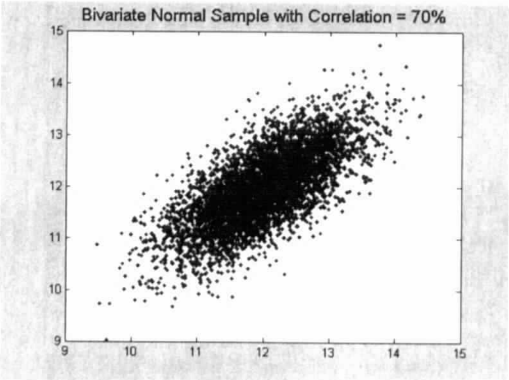
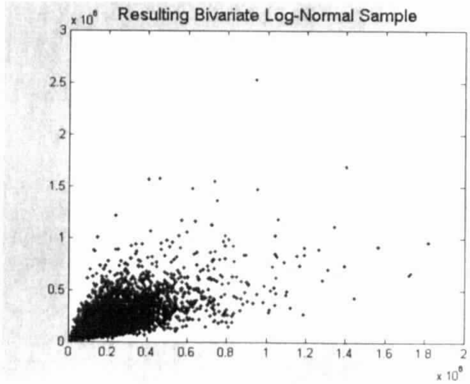


Figure 2



The sample correlation is 70.3%, showing the sample size is significant and the multivariate Normal generation algorithm is reasonably accurate. The correlation is also noticeable from the plot by the clustering of the points along the 45-degree line in an ellipsoid shape.

Figure 2 shows the plot of the bivariate log-Normal sample generated from the Normal sample by exponentiating every point. What is immediately apparent visually is the divergence of the points away from the 45-degree line. The divergence appears to grow wider as the magnitude of the generated loss amounts increases. This demonstrates the asymptotic tail independence of the bivariate log-Normal distribution.

Analytic measures of dependence fare no better. Apparently the simple act of exponentiating did not preserve the correlation, as the 70% sample correlation for the Normal sample drops to 64.3% for the log-Normal. Embrechts et al [2] explain why:

> "Linear correlation has the serious deficiency that it is *not* invariant under non-linear strictly increasing transformations." [2, Section 3.2]

If we perform this demonstration using more variables, the impact of the tail independence would be even more pronounced.

Other Copulas

Copulas are multivariate uniform-(0,1) distributions with a defined dependence relationship. Frees and Valdez [3] provide this definition:

> "*To define a copula, begin as you might in a simulation study by considering p uniform (on the unit interval) random variables, $u_1, u_2, \ldots, u_p$. Here, p is the number of outcomes you wish to understand. Unlike many simulation applications, we do not assume that $u_1, u_2, \ldots, u_p$ are independent; yet they may be related. This relationship is described through their joint distribution function*
>
> $$C(u_1, u_2, \ldots, u_p) = \Pr(U_1 \le u_1, U_2 \le u_2, \ldots, U_p \le u_p).$$
>
> *Here, we call the function C a copula.*" [3, p.2]

If the multivariate distribution is continuous, the copula is unique. Per Embrechts et al [2], if it is unique, the copula can be interpreted as the *dependence structure*. Since the multivariate Normal is continuous, its copula is unique and, therefore, the dependence structure is unique and completely defined by the linear correlation. If we are using the Normal copula, there is no way to generate any more tail dependence than we have seen. The asymptotic tail independence is a fundamental characteristic of the Normal copula itself, and makes it a poor choice for many simulation studies. If actuaries want different dependence relationships, they must employ *different copulas*.

Embrechts et al [2], Frees and Valdez [3] and Venter [5] discuss several promising alternative copulas. Many of the explanations are steeped in difficult statistical language that hampers the communication effort to broad actuarial audiences. To facilitate wider acceptance and use of these copulas in the North American actuarial community, actuaries need to become more familiar with alternative dependence measures. In addition, both algorithms and demonstration software need to be placed in the public domain.

Alternative Dependence Measures: other copulas require measures of dependence besides linear correlation: for example, rank correlation, Kendall's tau, and comonotonicity. See Embrechts et al [2] for an extensive discussion of these measures.

North American actuaries need to understand these new measures, how they are calculated, how they might be estimated from insurance data, how they measure tail dependence in particular, and how they compare with correlation. Of perhaps primary importance is "plain English" translations of the often complex formulas, to help actuaries develop an intuitive comfort level. Also critical are techniques that evaluate the appropriateness of various copulas for the particular study. Venter [5] presents several measures focusing on tail dependence, which is relevant to risk and capital measurement.

Algorithms and Software: Perhaps the Normal copula enjoys such widespread use in part because of its prevalence in so many software packages. Linear correlation can be calculated in a spreadsheet. Well-documented, widely available software implementations of new dependence measures and copulas would substantially increase their use and facilitate further research.

7. Conclusion

This paper has presented compelling evidence for alternative dependence models to the Normal copula. Many of the listed references provide detailed explanations of these models, but often from a statistical perspective that is difficult for a broad audience to grasp. There is a need for publication of survey papers to translate these often difficult statistical concepts into terms accessible to a broader audience. Equally important is the need for public domain demonstration software, giving practical examples of the measurement and use of these methods.

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Table 1
EPD Summary
29% Portfolio CV, 15% Correlation

Portfolio CV	28.6%
Correlation	15.0%

Average of Portfolio EPD	Contract EPD				
Contract Size ($000's)	20.0%	15.0%	10.0%	7.5%	5.0%
5,000	4.40%	1.56%	0.28%	0.07%	0.02%
10,000	4.80%	1.64%	0.28%	0.12%	0.01%
15,000	5.48%	1.79%	0.33%	0.10%	0.02%
25,000	5.95%	2.14%	0.51%	0.12%	0.02%
50,000	7.02%	3.01%	0.75%	0.33%	0.06%
75,000	8.50%	3.60%	1.13%	0.41%	0.09%
100,000	9.16%	4.55%	1.48%	0.62%	0.14%

Table 2
EPD Summary
32% Portfolio CV, 20% Correlation

Portfolio CV	32.1%
Correlation	20.0%

Average of Portfolio EPD	Contract EPD				
Contract Size ($000's)	20.0%	15.0%	10.0%	7.5%	5.0%
5,000	6.04%	2.78%	0.73%	0.17%	0.06%
10,000	6.02%	2.68%	0.67%	0.28%	0.04%
15,000	6.59%	2.75%	0.73%	0.25%	0.05%
25,000	6.98%	3.30%	0.81%	0.35%	0.08%
50,000	7.96%	3.73%	1.33%	0.51%	0.11%
75,000	9.14%	4.37%	1.54%	0.75%	0.16%
100,000	10.11%	5.15%	1.98%	0.85%	0.35%

Table 3
EPD Summary
36% Portfolio CV, 25% Correlation

Portfolio CV	35.7%
Correlation	25.0%

Average of Portfolio EPD	Contract EPD				
Contract Size ($000's)	20.0%	15.0%	10.0%	7.5%	5.0%
5,000	6.94%	3.67%	1.18%	0.32%	0.10%
10,000	7.45%	3.47%	1.23%	0.50%	0.12%
15,000	7.87%	3.67%	1.19%	0.56%	0.19%
25,000	8.29%	3.99%	1.29%	0.68%	0.10%
50,000	8.83%	4.36%	1.72%	0.71%	0.27%
75,000	9.57%	5.20%	2.14%	1.13%	0.29%
100,000	10.48%	5.79%	2.34%	1.26%	0.33%

Table 4
Ruin Summary
29% Portfolio CV, 15% Correlation

Portfolio CV	**28.6%**
Correlation	**15.0%**

Average of Portfolio Ruin	Contract Ruin				
Contract Size	**15.0%**	**12.5%**	**10.0%**	**7.5%**	**5.0%**
5,000	2.58%	1.42%	0.50%	0.12%	0.08%
10,000	2.76%	1.20%	0.48%	0.28%	0.00%
15,000	3.22%	1.26%	0.52%	0.16%	0.04%
25,000	3.38%	1.52%	0.90%	0.18%	0.02%
50,000	3.78%	1.98%	0.88%	0.48%	0.08%
75,000	4.86%	2.26%	1.32%	0.38%	0.10%
100,000	4.58%	2.94%	1.52%	0.66%	0.12%

Table 5
Ruin Summary
32% Portfolio CV, 20% Correlation

Portfolio CV	**32.1%**
Correlation	**20.0%**

Average of Portfolio Ruin	Contract Ruin				
Contract Size	**15.0%**	**12.5%**	**10.0%**	**7.5%**	**5.0%**
5,000	4.22%	2.60%	1.44%	0.30%	0.14%
10,000	3.94%	2.46%	1.20%	0.54%	0.12%
15,000	4.20%	2.32%	1.20%	0.42%	0.10%
25,000	4.68%	3.06%	1.22%	0.56%	0.12%
50,000	5.00%	2.92%	1.82%	0.74%	0.18%
75,000	6.00%	3.22%	1.98%	0.98%	0.22%
100,000	6.18%	4.02%	2.62%	0.94%	0.46%

Table 6
Ruin Summary
36% Portfolio CV, 25% Correlation

Portfolio CV	**35.7%**
Correlation	**25.0%**

Average of Portfolio Ruin	Contract Ruin				
Contract Size	**15.0%**	**12.5%**	**10.0%**	**7.5%**	**5.0%**
5,000	5.00%	4.06%	1.86%	0.58%	0.26%
10,000	5.34%	3.14%	2.32%	0.90%	0.24%
15,000	5.98%	3.58%	2.16%	0.94%	0.32%
25,000	6.40%	3.80%	1.86%	1.18%	0.22%
50,000	5.94%	3.70%	2.20%	1.02%	0.46%
75,000	6.84%	4.74%	2.94%	1.68%	0.42%
100,000	7.52%	5.04%	2.96%	1.76%	0.50%