Exam 8

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Exam 8

Advanced Ratemaking

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October 30, 2019

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 52.5 point examination consists of 19 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid/tape.

   - Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. For your Candidate ID number, four boxes are provided corresponding to one box for each digit in your Candidate ID number. If your Candidate ID number is fewer than 4 digits, begin in the first box and do not include leading zeroes. Your name, or any other identifying mark, must not appear.

   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.

   - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.

   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam, you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 13, 2019.

END OF INSTRUCTIONS

FORMULA SHEET

- For the negative binomial distribution (for use on question 3):
  - $f(x) = \binom{x+r-1}{x}(1-p)^r p^x$
  - $E[X] = \frac{pr}{1-p}$
- For the shifted Pareto distribution (for use on questions 7, 13 & 15):
  - $E[X] = \frac{\beta}{\alpha - 1}$
  - $E[X; x] = \frac{\beta}{\alpha-1}\left[1 - \left(\frac{\beta}{x+\beta}\right)^{\alpha-1}\right]$
  - $e_X(x) = \frac{x+\beta}{\alpha-1}$
  - $F_X(x) = 1 - \left(\frac{\beta}{x+\beta}\right)^{\alpha}$

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1. (2.25 points)

An insurer uses five geographical Storm Zones (A, B, C, D, and E) in their rating plan. The insurer refines their rating plan using the following approach:

- Divide each Storm Zone into numerous geographical sectors.
- For each geographical sector and storm type, calculate the observed storm frequency as number of storms per year.
    - There are three storm types defined as Moderate, Strong, and Severe storms.

The insurer performs two credibility procedures to produce estimates of the respective storm frequencies in each sector. The first is a multi-dimensional credibility procedure and the second is a single-dimensional credibility procedure (utilizing only a single storm type).

a. (1.5 points)

The multi-dimensional credibility results for a particular sector in Storm Zone C for Severe storms are given in the table below. The single-dimensional credibility factor is 0.30.

Storm Type	Coefficient	Storm Zone Mean Frequency	Sector Mean Frequency
Intercept	not provided		
Moderate	0.05	5.30	7.10
Strong	0.12	1.40	2.40
Severe	0.20	0.73	0.69

Calculate the mean frequency estimate of Severe storms for each of the two credibility procedures.

b. (0.75 point)

Fully explain why the single-dimensional credibility estimate in part a. above is lower than the multi-dimensional credibility estimate.

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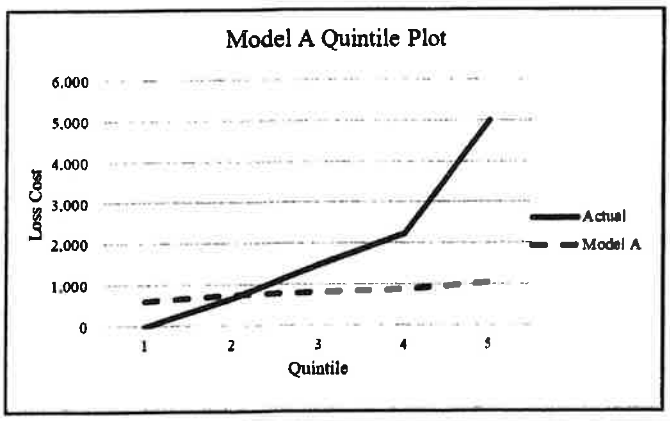
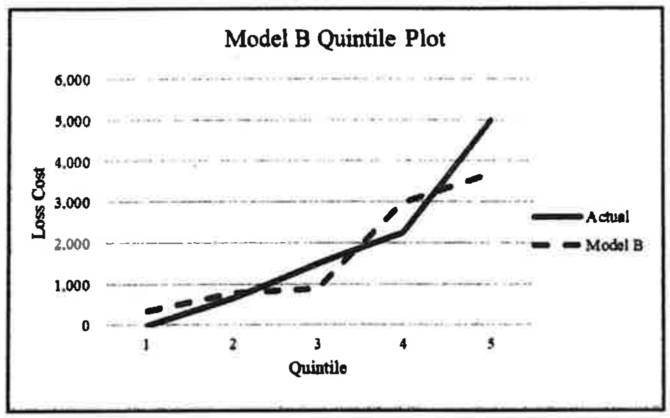
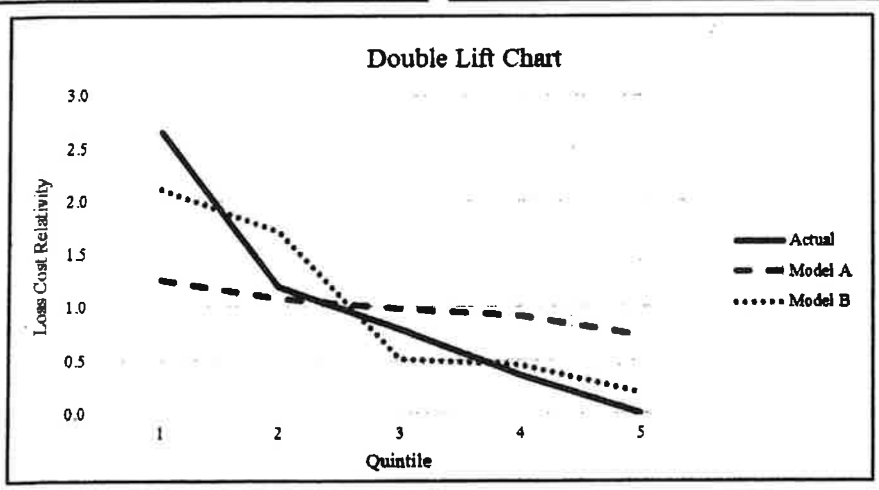
2. (2.75 points)

An actuary has built two generalized linear models to predict loss costs. Management has requested a series of model validation plots to demonstrate the appropriateness of each of the new models. Output for each model, simple quintile plots, and a double lift plot are shown below:

Observation	Actual Loss Cost	Model A Loss Cost	Model B Loss Cost	Earned Premium
1	1,500	825	900	1,800
2	675	765	800	1,450
3	0	615	350	2,375
4	2,250	900	3,000	2,625
5	5,000	1,050	3,700	4,875







Given the following:

- The actuary has already provided management with the simple quintile plots and the double lift chart shown above.
- The company has implemented several segmented rate changes over the last three years.

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a. (1 point)

For each model, provide a loss ratio plot that management can use to assess lift. Identify the basis of sorting the data.

b. (0.75 point)

Briefly describe one drawback of each type of model validation plot that the actuary has provided to management, including the plot produced in part a. above.

c. (1 point)

Using all three types of model validation plots provided to management, recommend which model should be implemented. Do not perform any calculations.

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3. (1.75 points)

An insurance company has a private passenger auto book of business with an experience modification factor in its rating plan.

Given the following:

- Annual claims for an individual driver follow a negative binomial distribution with $r = 10$.
- The expected claim frequency for the entire book of business is 0.101.
- The credibility for the group of risks that have had at least one accident in the last year is 0.02.

a. (1.25 points)

Calculate the experience modification factor for a policy that has had at least one accident in the last year.

b. (0.5 point)

Describe why a class with a higher volume of claims and more exposures may have less credibility than a class with fewer claims and exposures.

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4. (3 points)

An actuary wants to cluster six workers compensation classes based on excess ratios at two limits: 750,000 and 1,500,000. The actuary decides to use a weighted k-means algorithm with three clusters. Given the following:

Class	On-Leveled Earned Premium	Normalized Excess Ratio at 750,000 Limit	Normalized Excess Ratio at 1,500,000 Limit	Initial Cluster
1	7,000,000	0.250	0.095	A
2	8,000,000	0.200	0.080	B
3	7,500,000	0.170	0.062	C
4	6,000,000	0.130	0.048	A
5	5,000,000	0.450	0.217	A
6	8,500,000	0.240	0.098	B

- At the start of the algorithm, the actuary randomly assigned each class to a cluster.
- The actuary has observed that the 750,000 limit is selected by the insured 75% of the time, consistent across all six classes.

Briefly justify which measure to use to calculate the distance between vectors and using that measure, determine the cluster for each class after the first iteration of the weighted k-means algorithm.

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5. (2.75 points)

The following confusion matrix shows the result from a claim fraud model with a discrimination threshold of 25%:

	Predicted	
Actual	Yes	No
Yes	72	162
No	63	1203

a. (0.5 point)

Identify a link function that can be used for a generalized linear model that has a binary target variable and briefly explain why this link function is appropriate.

b. (0.5 point)

Calculate the sensitivity and specificity from the above data.

c. (1.5 points)

Plot the receiver operating characteristic (ROC) curve with the discrimination threshold of 25%. Label each axis, the coordinates, and the discrimination thresholds of 100%, 25%, and 0% on the curve.

In addition, plot the ROC curve for each of the following two models:

i. A model with no predictive power
ii. A hypothetical "perfect" model

d. (0.25 point)

Briefly describe how the severity of claims will impact the selection of the model threshold.

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6. (2 points)

An actuary creates a generalized linear model (GLM) to estimate commercial property claim frequency by occupancy class and amount of insurance (AOI) for sprinklered and non-sprinklered risks. Given the following:

- Occupancy class is a categorical variable with four levels: class 1, 2, 3 and 4.
- Sprinklered status is a categorical variable with two levels: sprinklered and non-sprinklered.
- The natural log of AOI, ln(AOI), is a continuous variable.
- The log link function is selected.
- An interaction variable is included as ln(AOI) for sprinklered and zero otherwise.
- The model results are as follows:

Parameter	**Coefficient**
Intercept	-8.4607
Occupancy class 2	0.2714
Occupancy class 3	0.3620
Occupancy class 4	0.0395
Sprinklered	0.7228
ln(AOI)	0.4311
Sprinklered:Yes, ln(AOI)	-0.0960

a. (0.75 point)

Calculate the ratio of the estimated model frequency of a sprinklered property to that of a non-sprinklered property for AOI = 200,000 and occupancy class 2.

b. (0.75 point)

Calculate the intercept term if AOI is centered at the base level of 200,000.

c. (0.5 point)

Briefly describe two advantages of centering variables of a GLM at their base levels.

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7. (7.5 points)

An actuary is pricing a workers compensation policy. Given the following:

Policy Effective Date	January 1, 2020
Policy Term	One year
Annual Loss Trend	4.5%
Cap for individual claims	$100,000
Credibility factor	0.40
Expected ultimate loss before modification	$1,064,000

As of June 30, 2019, ground up reported losses are:

Policy Year	Total Reported Loss	Individual Claims over $100,000	
2016	$392,457	$128,305	
2017	$1,013,863	$525,626	$152,860
2018	$459,798	$275,865	
2019	$181,325	None	

The following limited loss development factors (LDFs) apply to this policy:

Maturity to Ultimate	Limited LDF
42 months	1.052
30 months	1.094
18 months	1.286

Based on an internal study, the actuary believes that claim severity follows a shifted Pareto distribution with $\alpha = 1.3$ and $\beta = 22{,}800$.

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The following expenses apply to this policy:

Loss Adjustment Expense (LAE)	7.5% of loss
Taxes and Fees	3.5% of gross premium
Acquisition	17% of gross premium
Profit and Contingencies	0% of gross premium

a. (2.25 points)

Calculate the expected ground up reported loss limited to $100,000 per claim for this insured for the three years of experience combined.

b. (1.5 points)

Calculate the total modified ground up unlimited expected loss for this policy.

c. (0.25 point)

Alternatively, the actuary could have trended and developed reported losses to the cost level of the prospective policy period. Briefly explain why this approach would not produce an identical modification factor.

d. (0.5 point)

Calculate the guaranteed cost premium for this insured.

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The insurer's management is concerned about capacity on risks of this type. To address these concerns, they are requiring facultative reinsurance to support this account. Under the treaty, the reinsurer would assume all aggregate losses between \$2,000,000 and \$4,000,000. The primary insurer will retain all LAE.

The primary insurer's actuary believes the following Table M and the associated expected loss groupings (ELGs) are appropriate for risks of this type:

ELG	Loss Range
31	\$730,000 – 820,000
30	\$820,001 – 930,000
29	\$930,001 – 1,090,000
28	\$1,090,001 – 1,280,000
27	\$1,280,001 – 1,515,000
26	\$1,515,001 – 1,844,000

	ELG					
Entry Ratio	31	30	29	28	27	26
1.50	0.1876	0.1764	0.1649	0.1529	0.1442	0.1343
1.75	0.1490	0.1376	0.1257	0.1131	0.1048	0.0963
2.00	0.1195	0.1083	0.0964	0.0838	0.0762	0.0692
2.25	0.0968	0.0859	0.0745	0.0623	0.0555	0.0497
2.50	0.0791	0.0687	0.0579	0.0465	0.0404	0.0357
2.75	0.0652	0.0554	0.0453	0.0347	0.0295	0.0257
3.00	0.0541	0.0450	0.0356	0.0260	0.0215	0.0185
3.25	0.0452	0.0367	0.0282	0.0196	0.0157	0.0133
3.50	0.0380	0.0302	0.0224	0.0148	0.0115	0.0096
3.75	0.0321	0.0250	0.0179	0.0111	0.0084	0.0069
4.00	0.0273	0.0207	0.0144	0.0084	0.0062	0.0050

e. (1.5 points)

Calculate the loss expected to be ceded to the reinsurer under this treaty. Round to the nearest entry ratio (i.e., do not interpolate).

f. (1 point)

A reinsurer has quoted a premium of \$200,000 for this treaty. Calculate the premium the primary insurer must charge in order to maintain the same underwriting profit.

g. (0.5 point)

Explain how the assumptions of the primary insurer's actuary may have resulted in an inequitable premium calculated in part f. above.

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8. (1.5 points)

Given the following for a policy:

- Basic Limit = 100,000
- Expected Basic Limit Indemnity = 65,000
- Allocated Loss Adjustment Expense (ALAE) = 20% of Indemnity
- Unallocated Loss Adjustment Expense = 1.5% of Indemnity and ALAE
- All other expenses are variable and are 15% of the premium
- Risk load = 0%
- Profit load = 2.5%
- The following table shows Indemnity-only limited expected severity:

Limit L	E[X; L]
100,000	58,750
200,000	100,000
500,000	156,250
1,000,000	218,750

Calculate the premium for this policy with a 500,000 limit over a 500,000 deductible.

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9. (1 point)

An insurer's risk classification system contains four classes (A, B, C, and D). The insurer is considering using experience rating to rate all four classes.

Given the following:

	Low Variance of Hypothetical Means within Class	High Variance of Hypothetical Means within Class
Low Expected Value of Process Variance	Class A	Class B
High Expected Value of Process Variance	Class C	Class D

From the insurer's perspective, briefly explain the class for which experience rating is expected to be the most useful and the class for which experience rating is expected to be the least useful.

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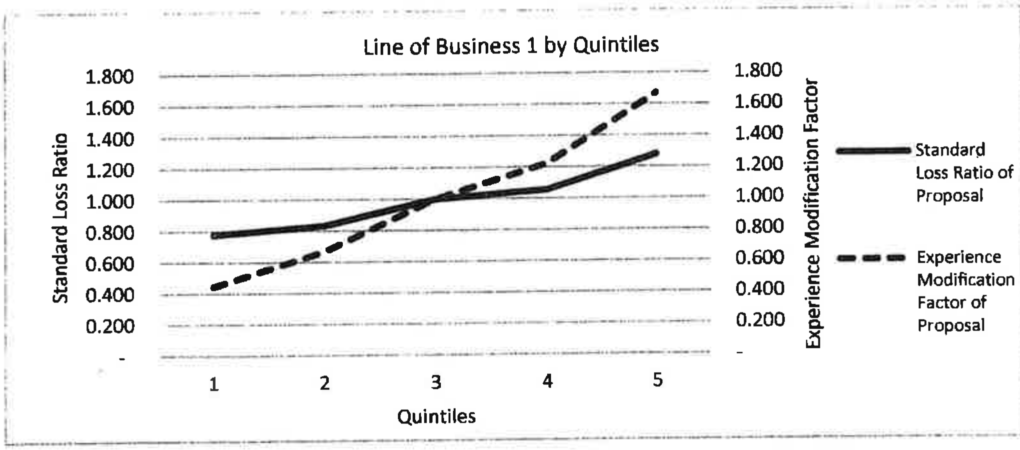
10. (1.5 points)

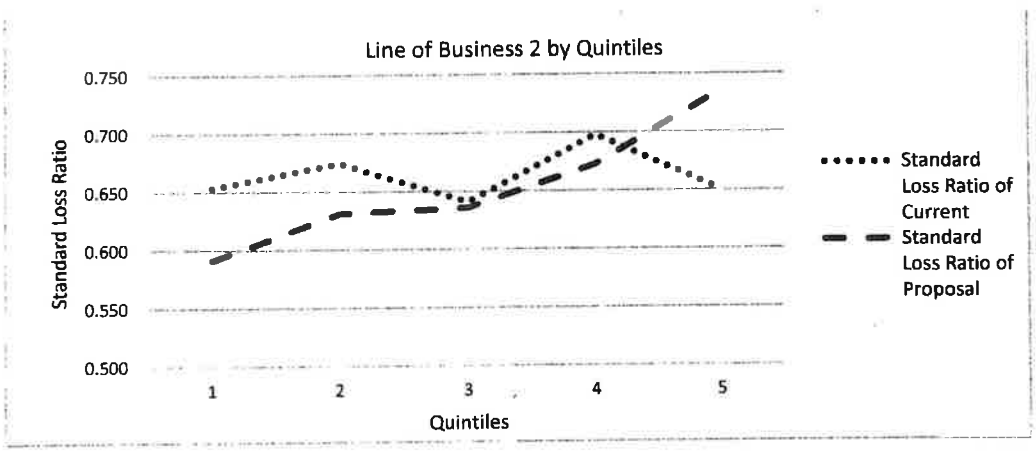
An actuary has created new models to calculate the experience modification factor for two lines of business.

Given the following:

- Line of business 1 (currently does not use experience rating)



- Line of business 2 (currently uses experience rating)



Line of Business 2	Efficiency Test
Current Plan	0.0346
Proposed Plan	0.1184

For both lines of business, assume that the cost of implementation is negligible.

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a. (0.5 point)

For line of business 1, evaluate whether the new experience modification should be implemented.

b. (0.5 point)

For line of business 1, describe a way to improve the results of the experience modification.

c. (0.5 point)

For line of business 2, evaluate whether the proposed plan should be used.

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11. (3.75 points)

Consider the following sample of five insurance risks which have been experience rated:

Risk	Mod	Manual Loss Ratio	Standard Loss Ratio
1	0.60	0.60	1.00
2	0.79	0.80	1.01
3	X	0.96	Y
4	1.10	1.10	1.00
5	1.14	1.15	1.01

The actual claims for Risk 3 in the experience period are as follows:

Claim #	Incurred Loss Amount
001	3,200
002	3,000
003	2,800
004	2,700
005	3,300
006	2,900
007	10,000
008	3,100
009	3,000
010	3,100
Total	37,100

- Expected Losses for Risk 3 in the experience period = 32,000
- Credibility factor = 0.75

a. (0.5 point)

Calculate the modification factor, X, for Risk 3.

b. (0.5 point)

Provide a recommendation to improve the effectiveness of experience rating for Risk 3 and briefly justify.

c. (2.75 points)

Show, quantitatively, that the recommendation in part b. above improves the plan. Assume no other risks are impacted by the recommendation.

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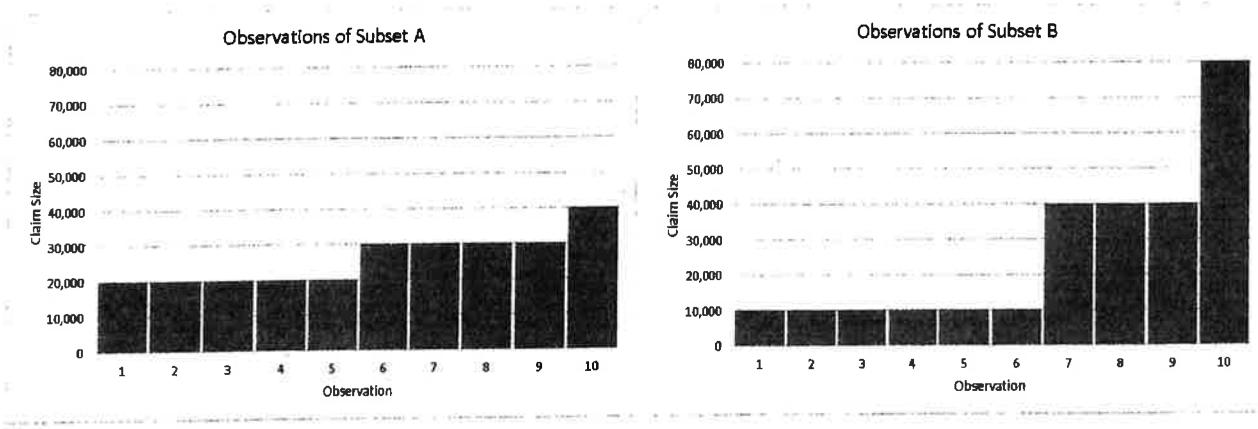
12. (2 points)

An actuary is assessing the current Table M underlying the pricing of a book of business. All policies within the book of business have identical expected losses and are expected to have exactly one claim during the policy period. The actuary divides the book of business into two subsets of policies, Subset A and Subset B, and finds the following distributions of aggregate losses:



a. (0.5 point)

Discuss why the current Table M may not be appropriate for this book of business.

b. (1 point)

Using the observations in Subset B, calculate the insurance charges associated with the following claim sizes:

i. 10,000
ii. 40,000
iii. 80,000

c. (0.5 point)

Briefly describe two considerations when using the Table M calculated in part b. above to support the creation of a Table M for a different line of insurance.

13. (2 points)

For a given policy that provides first dollar coverage, there is a 50% probability that no claims will occur during the policy period. Given that a claim does occur, the frequency distribution is shown below:

Number of Claims	Probability
1	70%
3	20%
5	10%

The claim-size variable X for this policy follows a shifted Pareto distribution with the following characteristics:

- $e_X(6{,}000) = 15{,}000$
- $E[X] = 3{,}000$
- $E[X; 4{,}000] = 1{,}433.30$

a. (1.5 points)

Calculate the expected aggregate loss in the layer 1,000 excess of 4,000 for this policy.

b. (0.5 point)

Explain one difficulty that can arise when trying to fit an aggregate loss distribution function to losses in an excess layer.

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14. (2 points)

An insured is written under a retrospective rating plan with a minimum and a maximum premium with the following parameters:

Loss Conversion Factor	1.07
Expected Loss	\$825,000
Expenses	\$20,000
$\varphi(r_G)$	0.40
$\varphi(r_H)$	0.80
r_G	1.21
r_H	0.50

a. (1 point)

Calculate the basic premium.

b. (1 point)

Upon renewal, the insured requests that a per occurrence claim limit of \$200,000 be added to the current policy. The pricing actuary estimates that, on average, the insured will have \$400,000 of losses that exceed this per occurrence limit during the next policy period. To calculate the expected increase to the basic premium, the actuary loads the \$400,000 for expenses and profit.

Fully discuss the appropriateness of the actuary's methodology.

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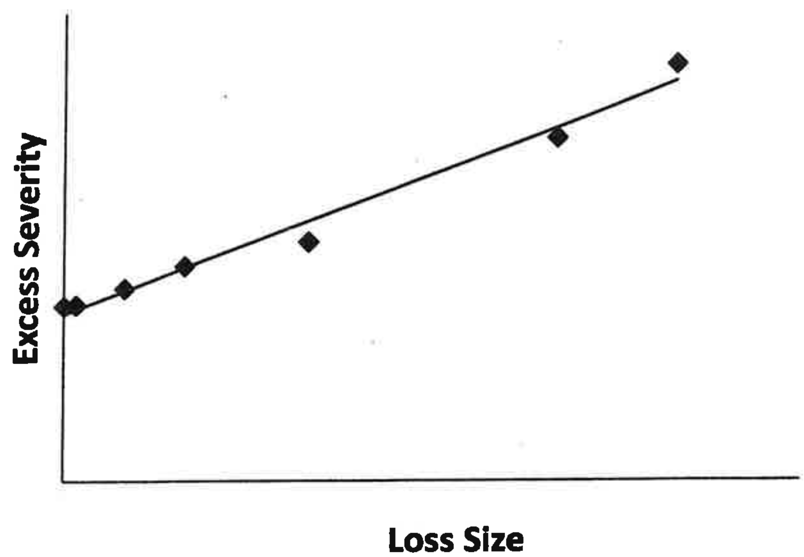
15. (3.5 points)

An actuary is pricing a retrospectively rated policy with a per occurrence limit of 20,000 and no minimum or maximum ratable loss.

Using claim data from a group of similar policies, the actuary fit the following regression line to the excess severity to estimate losses using the shifted Pareto distribution. The fitted regression function is $y = 0.4467x + 16112$ with $R^2 = 0.9826$.



Given the following:

		Included in:
Loss adjustment expenses as a percentage of loss	10%	Loss conversion factor
Commission as a percentage of retrospective premium	9%	Tax multiplier
Premium tax as a percentage of retrospective premium	8%	Tax multiplier
Fixed overhead expenses	50,000	Basic premium
Underwriting profit as a percentage of expected excess loss	6%	Basic premium

- The number of claims for ground up losses is Poisson distributed with λ =15.

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a. (2 points)

Calculate the basic premium.

b. (1 point)

Assuming that actual limited losses equal half the expected limited losses, calculate the retrospective premium.

c. (0.5 point)

Assess the actuary's decision to use the shifted Pareto distribution to estimate excess severity.

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16. (3 points)

The loss experience of five similarly sized risks is shown in the table below:

Risk	Aggregate Unlimited Loss	Aggregate Limited Loss
1	30,000	30,000
2	95,000	60,000
3	110,000	90,000
4	160,000	60,000
5	415,000	360,000
Total	810,000	600,000

Limited losses have a per-occurrence limit of 50,000 applied.

a. (1.5 points)

Construct a Limited Loss Table M based on the experience above, displaying entry ratios ranging from 0 to 3 in increments of 0.5 and the corresponding insurance charges.

b. (1.5 points)

An insured similar to the risks above is evaluating three potential policies:

1. A guaranteed cost policy
2. A large dollar deductible policy with a 50,000 per-occurrence limit
3. A retrospectively rated policy with an underlying 200,000 aggregate limit and no per-occurrence limit

Given the following:

- The insured would like to minimize cost
- Maintaining a stable, predictable cash flow is a priority for the insured
- The insured has recently implemented a successful claim severity reduction program

Briefly discuss one advantage and one disadvantage of each of the three policies from the perspective of this insured.

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17. (3 points)

An insurer is deciding on a proportional reinsurance strategy for the upcoming two years. A reinsurer proposes the following sliding-scale commission with a carry-forward provision. The contract will be identical for the second year.

Minimum Commission:	12.5% at an 80% loss ratio
Sliding 1:1 to	27.5% at a 65% loss ratio
Sliding .5:1 to a Maximum	35% at a 50% loss ratio

Assume the insurer places no value on the carry-forward provision after the second year, and ignore the time value of money (i.e., assume a 0% interest rate).

The following table details the expected loss distribution of the underlying business:

Range of Loss Ratios	Average Loss Ratio in Range	Probability of Being in Range
0% - 50%	43%	0.15
50% - 60%	57%	0.25
60% - 80%	68%	0.45
80% or above	85%	0.15

Calculate the reinsurer's expected technical ratio for each year if the insurer buys the sliding-scale commission contract for both years.

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18. (5.5 points)

A property insurer has an existing Quota Share reinsurance treaty in place. The insurer would like to further reduce its net loss exposure by exploring a proposed Property Per Risk Excess reinsurance treaty. Given the following:

Existing Quota Share Reinsurance Treaty

- 25% Quota Share
- 15% Ceding Commission

Proposed Property Per Risk Excess Reinsurance Treaty

- Subject premium of $100 million net of the Quota Share reinsurance treaty
- Covers net losses in excess of $500,000, up to a limit of $500,000
- The Quota Share reinsurance treaty will inure to the benefit of this treaty
- Ceded Loss Ratio for this Excess layer is estimated to be 90%

The table below illustrates the insurer's historical experience data net of the Quota Share that is subject to the proposed Property Per Risk Excess layer:

Historical Accident Year	On Level Subject Earned Premium ($ millions)	On Level Trended Ultimate Subject Loss Ratio	On Level Trended Ultimate Layer Loss Cost
2014	100	75%	25%
2015	120	75%	30%
2016	150	90%	45%
2017	80	75%	24%
2018	100	80%	36%

a. (0.5 point)

Calculate the loss cost of the Property Per Risk Excess layer using all 5 years of historical experience data provided.

CONTINUED ON NEXT PAGE

b. (1 point)

Assume that the following exposure curve definition applies to the Property Per Risk Excess layer:

$$G(x) = \frac{1 - 0.32428^{x}}{1 - 0.32428}$$

where $G(x)$ represents the ratio of pure risk premiums retained by the insurer, and x represents the ground up loss normalized to the maximum possible loss of $1 million.

Calculate the loss cost of the Property Per Risk Excess layer retained by the reinsurer based on the exposure curve.

c. (0.5 point)

Calculate the loss cost of the Property Per Risk Excess layer using a blend of the experience loss cost and the exposure loss cost based on a credibility weight of 80%.

d. (2 points)

The insurer's historical gross expenses are estimated to be 15% of historical gross premiums.

Calculate the insurer's expected net underwriting profit after application of both the existing Quota Share reinsurance treaty and the proposed Property Per Risk Excess reinsurance treaty.

The insurer is also exploring the following proposed modifications to the Quota Share reinsurance treaty:

- Option 1: Profit Commission equal to 100% of reinsurer profit above a 5% reinsurer margin
- Option 2: 20% Ceding Commission

e. (1 point)

Calculate the 5 year weighted average ratio of profit commission to ceded premium for the Quota Share reinsurance treaty using the historical experience data provided.

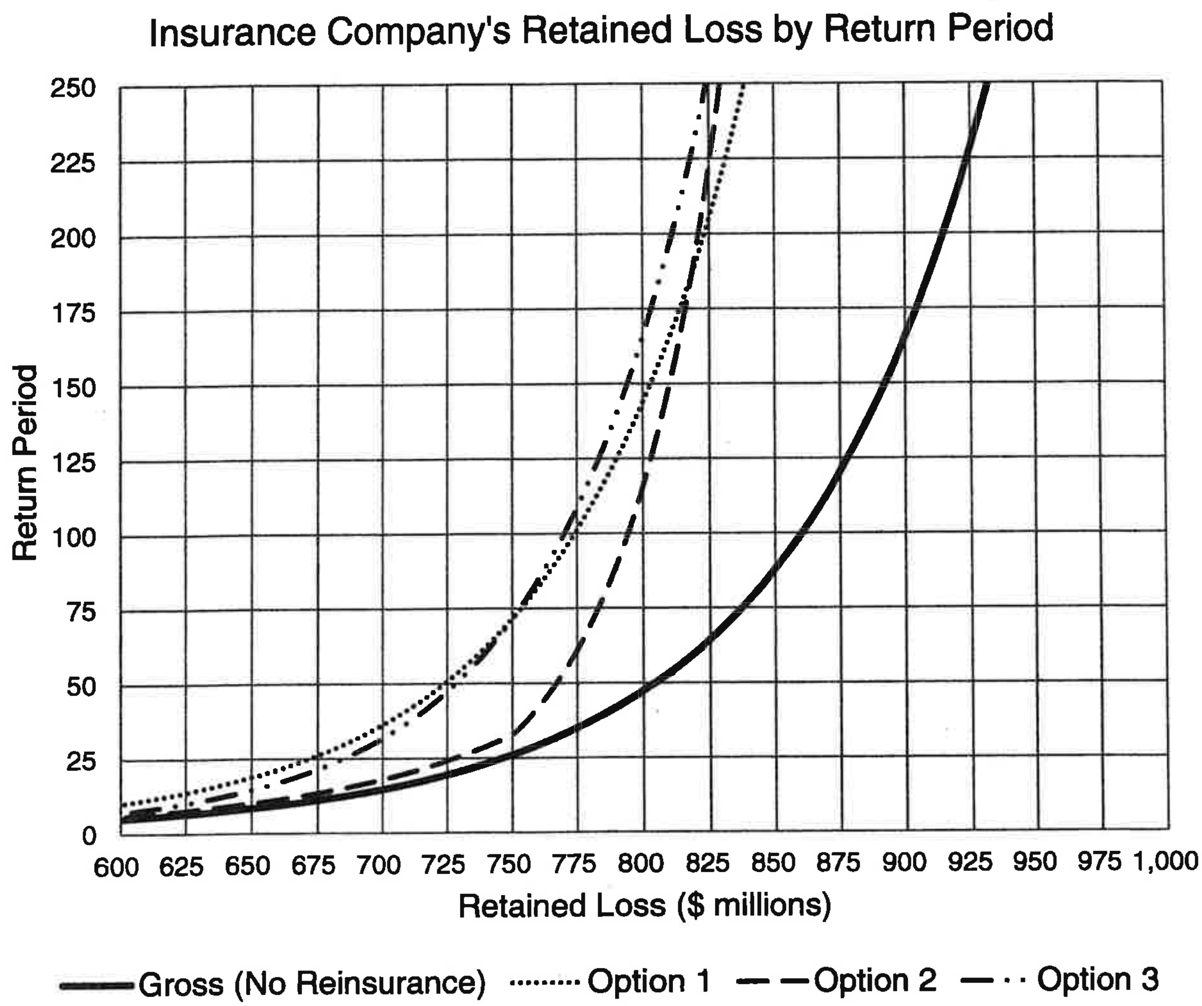
f. (0.5 point)

Briefly describe one advantage and one disadvantage of implementing the proposed profit commission versus the higher ceding commission from the insurer's perspective.

CONTINUED ON NEXT PAGE

19. (1.75 points)

An insurance company is deciding between three reinsurance treaty options from the same reinsurance company based on the following output from a catastrophe model:



Insurance Company	Gross (No Reinsurance)	Option 1	Option 2	Option 3
Retained AAL	\$500 million	\$450 million	\$486 million	\$488 million
Coefficient of Variation of Retained Loss	25%	25%	24%	22%
Coefficient of Variation of Ceded Loss	n/a	25%	76%	169%

CONTINUED ON NEXT PAGE

a. (0.5 point)

Calculate the probability that the insurance company retains more than $750 million of loss on a gross basis.

b. (0.75 point)

One of the options represents a Quota Share reinsurance treaty. Justify which of the three options it is, and state the Quota Share percentage.

c. (0.5 point)

Describe a reason why the reinsurance premium for Option 3 may be higher compared to Option 1.

END OF EXAMINATION

Exam 8
Advanced Ratemaking

POINT VALUE OF QUESTIONS

QUESTION	TOTAL POINT VALUE OF QUESTON	SUB-PART OF QUESTION (a)	(b)	(c)	(d)	(e)	(f)	(g)
1	2.25	1.50	0.75					
2	2.75	1.00	0.75	1.00				
3	1.75	1.25	0.50					
4	3.00	3.00						
5	2.75	0.50	0.50	1.50	0.25			
6	2.00	0.75	0.75	0.50				
7	7.50	2.25	1.50	0.25	0.50	1.50	1.00	0.50
8	1.50	1.50						
9	1.00	1.00						
10	1.50	0.50	0.50	0.50				
11	3.75	0.50	0.50	2.75				
12	2.00	0.50	1.00	0.50				
13	2.00	1.50	0.50					
14	2.00	1.00	1.00					
15	3.50	2.00	1.00	0.50				
16	3.00	1.50	1.50					
17	3.00	3.00						
18	5.50	0.50	1.00	0.50	2.00	1.00	0.50	
19	1.75	0.50	0.75	0.50				
TOTAL	52.50							

SAMPLE ANSWERS AND EXAMINER'S REPORT

FALL 2019 EXAM 8 EXAMINER'S REPORT

The Syllabus and Examination Committee has prepared this Examiner's Report as a tool for candidates preparing to sit for a future offering of this exam. The Examiner's Report provides:

- A summary of exam statistics
- General observations by the Syllabus and Examination Committee on candidate performance
- A question-by-question narrative, describing where points were commonly achieved and missed by the candidates.

The report is intended to provide insight into what the graders for each question were looking for in responses that received full or nearly-full credit. This includes an explanation of common mistakes and oversights among candidates. We hope that the report aids candidates in mastering the material covered on the exam by providing valuable insights into the differences between responses that are comprehensive and those that are lacking in some way.

Candidates are encouraged to review the Future Fellows article from June 2013 entitled "Getting the Most out of the Examiner's Report" for additional insights.

We hope that the details by question provided throughout this Examiner's Report will be helpful to future candidates.

EXAM STATISTICS:

- Number of Candidates: 1,080
- Available Points: 52.5
- Passing Score: 37
- Number of Passing Candidates: 376
- Raw Pass Ratio: 34.81%
- Effective Pass Ratio: 37.01%

In recognition that the length of the exam may have negatively impacted the performance of candidates, an aggregate downward adjustment was made to the pass score, determined based on various metrics.

GENERAL COMMENTS:

- Candidates should note that the instructions to the exam explicitly say to show all work; graders expect to see enough support on the candidate's answer sheet to follow the calculations performed. While the graders made every attempt to follow calculations that were not well-documented, lack of documentation may result in the deduction of points where the calculations cannot be followed or are not sufficiently supported.

SAMPLE ANSWERS AND EXAMINER'S REPORT

- Integrative Questions (IQs) were first introduced to Exam 8 in 2017 and are being used to test candidates' ability to apply and synthesize multiple advanced ratemaking ideas in addressing complex business problems. Both IQs this sitting were based on real-world scenarios and were designed to test multiple syllabus learning objectives at higher cognitive (Bloom's) levels. Candidates should expect to encounter similar sorts of questions in future sittings.
- Candidates are reminded of the following excerpt from the Exam 8 syllabus: "The ability to apply ratemaking knowledge and experience may be tested through questions dealing with problems for which there are no generally recognized solutions. The readings for Exam 8 should be studied for illustration of basic principles and theories, as well as for insight into advanced ratemaking problems and their solutions." This applies not only to Integrative Questions, but to the entire exam overall.
- Incorrect responses in one part of a question did not preclude candidates from receiving up to full credit for correct work on subsequent parts of the question that depended upon that response. This includes situations where candidates could not calculate an answer but made a reasonable one up in order to make further progress on the later part(s) of the question.
- Candidates should be cognizant of the way an exam question is worded. They must look for key words such as "briefly" or "fully" within the problem. We refer candidates to the Future Fellows article from December 2009 entitled "The Importance of Adverbs" for additional information on this topic.
- Candidates should note that the sample answers provided in the examiner's report are not an exhaustive representation of all responses given credit during grading, but rather the most common correct responses.
- In cases where a given number of items were requested (e.g., "three reasons" or "two scenarios"), the examiner's report often provides more sample answers than the requested number. The additional responses are provided for educational value, and would not have resulted in any additional credit for candidates who provided more than the requested number of responses. Candidates are reminded that, per the instructions to the exam, when a specific number of items is requested, only the items adding up to that number will be graded (i.e., if two items are requested and three are provided, only the first two are graded).
- Candidates are reminded that the syllabus for this exam states: "The CAS Syllabus & Examination Committee emphasizes that candidates are expected to use the readings cited in this Syllabus as their primary study materials." Based on candidate performance on certain questions, this does not appear to be the case. As an example, as noted in the Examiner's Report for the 2018 exam, it appears as if some candidates may not have reviewed the Case Study included as part of the Syllabus. Candidates are strongly encouraged to download this Excel file and work through all of the tabs of that file.

SAMPLE ANSWERS AND EXAMINER'S REPORT

- It should be noted that all exam questions have been written and graded based on information included in materials that have been directly referenced in the official Syllabus, which is located on the CAS website. The CAS takes no responsibility for the content of supplementary study materials and/or manuals produced by outside corporations and/or individuals that are not directly referenced in the official Syllabus.

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 1	
TOTAL POINT VALUE: 2.25	**LEARNING OBJECTIVE: A2**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

multi-dimensional:

0.73 + 0.2 (0.69 – 0.73) + 0.12 (2.4 – 1.4) + 0.05 (7.1 – 5.3) = 0.932

single-dimensional:

0.3 (0.69) + 0.7 (0.73) = 0.718

Sample 2

VH + b (VZ – VH) + c (WZ – WH) + d (XZ – XH)
severe cred-weighted estimate using multi-cred:
= .73 + .2 (.69 - .73) + .12 (2.4 – 1.4) + .05 (7.1 – 5.3)
= .932

using single dim cred:
.3 * .69 + (1 - .3) * .73
= .718

Part b: 0.75 point

Sample 1

Single dimensional is lower because the sector's experience for severe claim frequency is lower than that of the storm zone. However, multi-dimensional credibility is based on the theory that claim frequency for different storm types are related. Since both the moderate & strong storm types have higher frequency experience in the sector compared to the zone, the multi dim credibility is higher than the single dim.

Sample 2

Due to correlations with other storm types, strong and moderate, the sector freq. is greater than the zone freq., the multi-dim uses this info and calculates a higher est. than single cred., which does not use the correlation info.

Sample 3

Weight is given to the relative mean frequency of the sector to the storm zone, and the mean frequencies of the sector for the other two storm types are much higher than the storm zone mean frequencies. This will increase the estimate using multi-dimensional credibility because it assumes high frequencies in other storm types are correlated positively with future frequency for severe storms.

EXAMINER'S REPORT

Candidates were expected to understand multi-dimensional credibility and how it differs from single-dimensional credibility, to apply the relevant formulas to calculate credibility-weighted estimates of a particular quantity, and to discuss the drivers of each estimate.

Part a

Candidates were expected to correctly calculate both the multi-dimensional and single-dimensional credibility estimates for severe storm frequency in a given sector within storm zone C.

Common mistakes included:

- Reversing the sector and the storm zone
- Applying the wrong "intercept" in the multi-dimensional calculation (0.69 instead of 0.73)
- Calculating relativities to another storm type – or to the total frequency – within the sector and the zone separately before applying the formulas
- Using 0.2 instead of 0.3 for the single-dimensional credibility to be assigned to severe storm types in the sector.

Part b

Candidates were expected to provide a detailed rationale for why the multi-dimensional credibility estimate was larger than the single-dimensional credibility estimate.

In order to get full credit, candidates needed to express three primary ideas:

- Storm types are correlated
- Moderate and strong storm types have higher frequency than severe storms
- Moderate and strong storm frequency in the sector is larger than the corresponding frequencies in the storm zone.

Common mistakes included:

- Listing only one reason; in particular, simply noting that severe storm frequencies are likely correlated with moderate and strong storm frequencies
- Omitting a comparison or contrast between the sector frequencies and the storm zone frequencies.

QUESTION 2	
TOTAL POINT VALUE: 2.75	**LEARNING OBJECTIVE(S): A.4**
SAMPLE ANSWERS	
Part a: 1 point	

Sample 1

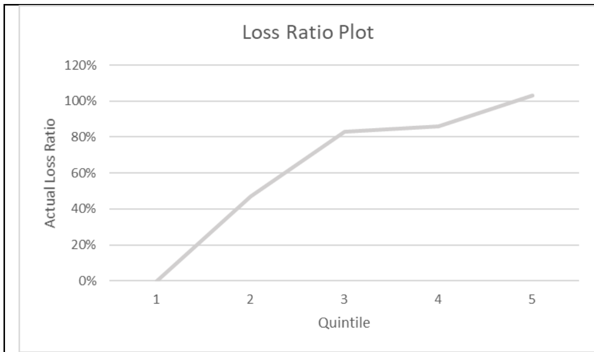
Observation	Actual LR
3	0
2	0.47
1	0.83
4	0.86
5	1.03

The data is sorted by modeled loss cost, ascending. The loss ratio plot will be the same for both models as they rank/put the observations in the same order.

Sample 2

Observation	Actual LR
3	0%
2	47%
1	83%
4	86%
5	103%

Same sort order so plot is the same. Sorted by model A and B predicted loss costs.



Sample 3

Observation	Actual LR
3	0
2	0.47
1	0.83
4	0.86
5	1.03

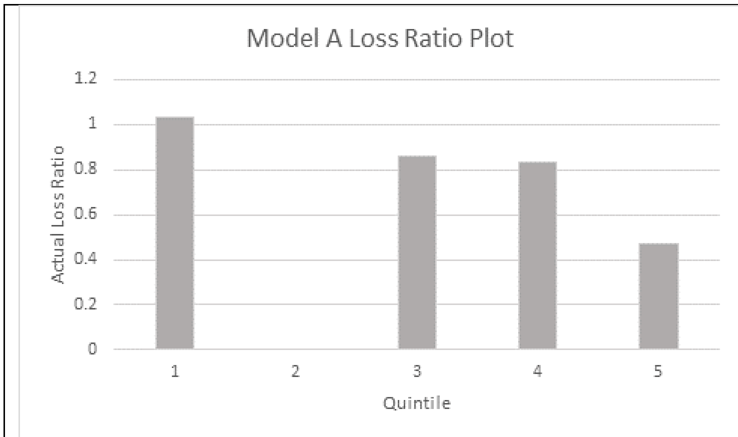
Sort by the model prediction (loss cost here) in ascending order. Models A and B have the same sort.

Sample 4

Sort by model predicted loss ratio for model A and B.

For Model A:

Observation	Predict LR	Actual LR
5	.22	1.03
3	.26	0
4	.34	.86
1	.46	.83
2	.53	.47



For Model B:

Observation	Predict LR	Actual LR
3	.15	0
1	.50	.83
2	.55	.47
5	.76	1.03
4	1.14	.86

Part b: 0.75 point

Sample 1

- The two LR charts look the same because when sorted, Model A & B have the same ordering
- The double lift chart is difficult to explain to management
- We need to produce two quintile plots to compare A & B, so it is more work

Sample 2

- LR Chart – only can tell how well each model identifies differences in risks, not if the predictions are accurate
- Quantile Plot – graphs are on separate charts and can only compare by looking at 2 charts
- Double Lift – compares where model A disagrees with model B most (since sorted based on this ratio) so can be harder to interpret

Sample 3
- Quintile: Need to sort groups into quintiles with approximately equal exposures which could be difficult. If there are a few large risks and many small risks, results could be skewed
- Double Lift: the sort order of model A / model B is unintuitive
- Loss Ratio: only assesses how well the GLM differentiates risks; no validation of predicted losses

Sample 4
- Quintile plot is less of a direct model comparison because it requires separate plots for each model
- Double lift charts are harder for business partners to interpret
- The LR data (EP) must be on-leveled and it may not have been here

Sample 5
- Quintile Plots: Model A output much lower loss costs, which likely wouldn't be implemented. This makes for an unfair comparison to model B
- Double Lift Chart: This plot normalizes everything and ignores that model A output lower loss costs in aggregate
- Loss Ratio Plot: This shows how well the model does at identifying risk but not at model performance

Sample 6
- Quintile Plot: Does not normalize predictions, which can make it difficult to compare one model to another
- Double Lift Plot: Does not provide information about actual loss dollars
- Loss Ratio Plot: Does not provide actual model predictions; only the order. Also not clear what basis earned premium is on. If it's not on-level, this chart can be misleading.

Part c: 1 point

Sample 1
- Based on simple quintile plot, model B is much better at predicting actual loss cost (two lines are closer)
- Based on double lift chart, model B line is also better at predicting the actual loss cost (model B & actual line are closer than model A & actual)
- Based on loss ratio chart, both models perform equally at segmenting good and bad risks
- I recommend Model B

Sample 2
- Single Quintile Plot: Model B has a better match for the model loss cost and actual loss cost
- Double Lift Chart: Model B has a better match for the model B loss cost and actual loss cost
- Loss Ratio Plot: There is upward trend in model B plot indicating it outperforms the current rating plan
- I would recommend management implement model B

Sample 3

- The loss ratio plots for both models are identical but do indicate the models successfully recognize the differences between the risks
- The quintile and double lift charts show that while model A is monotonically increasing, the model B has greater predictive accuracy
- I would recommend implementing model B

EXAMINER'S REPORT

Candidates were expected to create and assess validation plots that can be used to compare models. Candidates were also expected to understand shortcomings of model validation plots.

Part a

Candidates were expected to create loss ratio plots for Model A and Model B. The syllabus reading states that observations should be sorted by model prediction (which this question stated was loss cost) when creating loss ratio plots. Sorting by predicted loss ratio was an acceptable full credit alternative but required additional work and is not necessarily superior to sorting by predicted loss cost.

Common mistakes included:

- Sorting observations by something other than predicted loss cost or predicted loss ratio
- Plotting predicted loss ratio instead of actual loss ratio
- Calculation errors when determining the actual and/or predicted loss ratios
- Mislabeling or forgetting to label plots.

Part b

Candidates were expected to explain a drawback of simple quintile plots, double lift plots, and loss ratio plots.

Common mistakes included:

- Providing drawbacks of model validation plots in general rather than for one of the specific plots
- Discussing what is shown on each plot without providing an explanation of an actual drawback or shortcoming
- Discussing drawbacks of implementing the Model A or Model B instead of more general drawbacks of the model validation plots.

Part c

Candidates were expected to assess Model A and Model B using the simple quintile plots, double lift chart, and the loss ratio charts produced in part a. Candidates were also expected to use this assessment to recommend a particular model.

Common mistakes included:

- Stating that a plot showed that a model had more lift without any additional explanation
- Using only one or two of the model validation plots instead of all three types
- Comparing the vertical distance between the first and last quintiles of the modeled loss costs instead of the actual loss costs in the simple quintile plots
- Comparing actual loss ratios to modeled loss ratios in the loss ratio plot

- Discussing the relative performance of each model without providing a final recommendation.

QUESTION 3	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): A1**
SAMPLE ANSWERS	
Part a: 1.25 points	

Sample 1

z = 0.02 r = 10

$.101 = \frac{10p}{1-p}$

$.101 - .101p = 10p$

$p = .01$

$\Pr(N=0) = \binom{9}{0}(1-.01)^{10}(.01)^{0}$

$\Pr(N=0) = .9044$

$R = \frac{1}{1-\Pr(N=0)}$

$R = \frac{1}{1-.9044} = 10.458$

$Mod = (.02)(10.458) + (1-.02)$

$Mod = 1.1892$

Sample 2

$E(x) = \frac{pr}{1-p}$

$.101 = \frac{p*10}{1-p}$

$1 - p = 99p$

$p = .01$

$R = \frac{1}{1-(1-.01)^{10}}$

$R = 10.458$

$Mod = Z*R + (1-Z)$

$Mod = (.02)(10.458) + (1-.02)$

$Mod = 1.18916$

Sample 3

$\frac{pr}{1-p} = 0.101 = \frac{10p}{1-p}$

$0.101 - 0.101p = 10p$

$p = .009999$

$(1-p)^{10} = (1-.009999)^{10} = 0.904391119$

$$E(x|x \geq 1) = \frac{0.101}{1 - .90439119} = 1.056387$$

$$Mod = \frac{(.02)(1.056387)+(1-.02)(0.101)}{0.101} = 1.189$$

Part b: 0.5 point

Sample Answers:

- Experience rating credibility depends not only on volume of data but also the variance within a class. Therefore, a class may receive more credibility than a class with more volume if it has more variance within the class.

- Experience rating is meant to distinguish an individual within the class. If there is low variance within a class, then experience rating is not as useful, so credibility is lower, even if the class has high volume.

- If a class is very homogeneous already, experience will not actually be very useful and will have a low credibility.

- If a class is well-defined the experience has less credibility than if a class is less defined even if the well-defined class has more claims and exposure.

EXAMINER'S REPORT

Candidates were expected to be able to calculate an experience mod based on a claim count distribution and to be able to describe what determines the credibility given to experience rating for a class.

Part a

Candidates were expected to calculate the negative binomial parameter p, calculate R, and then calculate the experience mod.

Common mistakes included:

- Assuming p = E(x) = .101
- Assuming claim counts followed a Poisson distribution
- Using an incorrect formula for R.

Part b

Candidates were expected to understand that credibility for experience rating depends not only on the volume of data in the experience period but also on the amount of variation of individual hazards within the class. Candidates were also expected to understand that more credibility is given to experience rating when there is more variation within the class.

Some candidates said credibility would be lower because risks may be entering or leaving the class, or risk characteristics within the class might be changing. This is a reason credibility does not increase directly with the number of years of experience as the increase in volume of data alone would suggest, but it was not accepted as a reason credibility given to experience rating would differ between two classes.

Common mistakes included:

- Stating that more variation would produce less credibility
- Discussing variation in the claim count distribution rather than variation of individual risks within the class.

QUESTION 4	
TOTAL POINT VALUE: 3	LEARNING OBJECTIVE(S): A1
SAMPLE ANSWERS	

Sample 1:

Recommend L^1:

- Class 5 looks to be an outlier, and L^2 will penalize this outlier more | selection of limit being consistent across classes means that L^1 can be used
- Minimize absolute error or relative error

Sample 2:

Recommend L^2:

- Class 5 is not enough to be considered an outlier, okay to use L^2. Or could say that the limits are not equally likely to be selected by insureds so L^1 error is not minimized.
- Minimize mean square error or variance between clusters
- It is the more traditional approach so use since it doesn't make a significant difference as per the paper
- Better reflects a skewed distribution

Determine the centroid of each initial cluster:

R_A(750K) = (7000 * 0.25 + 6000 * 0.13 + 5000 * 0.45) / (7000 + 6000 + 5000) = 0.266

R_A(1.5M) = (7000 * 0.095 + 6000 * 0.048 + 5000 * 0.217) / (7000 + 6000 + 5000) = 0.113

R_B(750K) = (8000 * 0.2 + 8500 * 0.24) / (8000 + 8500) = 0.221

R_B(1.5M) = (8000 * 0.08 + 8500 * 0.098) / (8000 + 8500) = 0.089

R_C(750K) = 0.17

R_C(1.5M) = 0.062

Sample 1 (Using L^1 Measure, even weights by limit):

Class	Distance to R_A	Distance to R_B	Distance to R_C	New Cluster

1	0.034 = abs((0.25-0.266)) + abs((0.095-0.113))	0.035	0.113	A
2	0.099	0.030	0.048	B
3	0.147	0.078	0.000	C
4	0.201	0.132	0.054	C
5	0.288	0.357	0.435	A
6	0.041	0.028	0.106	B

Sample 2 (Using L^2 Measure, even weights by limit):

Class	Distance to R_A	Distance to R_B	Distance to R_C	New Cluster
1	0.024 = [(0.25 - 0.266)^2 + (0.095 - 0.113)^2]^(1/2)	0.030	0.087	A
2	0.074	0.023	0.035	B
3	0.109	0.058	0.000	C
4	0.151	0.100	0.042	C
5	0.211	0.262	0.320	A
6	0.030	0.021	0.079	B

Sample 3 (Using L^1 Measure, 75/25 weights):

Class	Distance to R_A	Distance to R_B	Distance to R_C	New Cluster
1	0.016 =abs(.75*(0.25-0.266)) +abs(.25*(0.095-0.113))	0.023	0.068	A
2	0.057	0.018	0.027	B
3	0.084	0.045	0.000	C
4	0.118	0.078	0.034	C
5	0.164	0.204	0.249	A
6	0.023	0.017	0.062	B

Sample 4 (Using L^2 Measure, 75/25 weights):

Class	Distance to R_A	Distance to R_B	Distance to R_C	New Cluster

1	0.016 = [.75*(0.25 - 0.266)^2 + .25*(0.095 - 0.113)^2]^(1/2)	0.026	0.071	A
2	0.059	0.018	0.027	B
3	0.087	0.046	0.000	C
4	0.122	0.081	0.035	C
5	0.168	0.209	0.255	A
6	0.023	0.017	0.063	B

EXAMINER'S REPORT

The candidates were expected to explain which distance measure should be used and how to calculate the means square error and clustering process.

Common mistakes included:

- Not adequately explaining which measure to use
- Not taking the square root for L^2
- Explaining the choice to weight with 75/25 rather than explaining the choice of L^1 or L^2
- Weighting the centroids together and then weighting the distance together: mathematically this gives the same answer when using L^1 measure but does not make sense when using the L^2 measure as it is simply the difference in this case
- Using only one limit instead of both limits to measure distance
- Not calculating all classes and not fully explaining how to compare the distances that weren't calculated.

QUESTION 5	
TOTAL POINT VALUE: 2.75	**LEARNING OBJECTIVE(S): A3, A4**
SAMPLE ANSWERS	
Part a: 0.5 point	

Sample 1

Logit link function $= \ln\left(\frac{\mu}{1-\mu}\right)$; 0≤μ≤1

Appropriate because the inverse of the logit function is the logistic function. Logistic function produces a variable between 0 and 1 which coupled with a discrimination threshold produces a binary 0 or 1 outcome.

Sample 2

The logit link function is appropriate because its inverse is the logistic which is $f(x) = \frac{1}{1+e^{\wedge}}$. This allows us to take an unbounded value of x and return a value between 0 and 1, which is what we want when estimating probabilities.

Sample 3

The logit link function is used for logistic regression in GLM. It is appropriate b/c it has the ability to map any number from "-∞ to ∞" to "0 to 1". We can then pick a threshold like 50%. Then if we get .39 which is below 50% we can assign a "No". If greater than 50%, then we can assign "Yes".

Sample 4

Logit function $g(x) = \ln\left(\frac{\mu}{1-\mu}\right)$

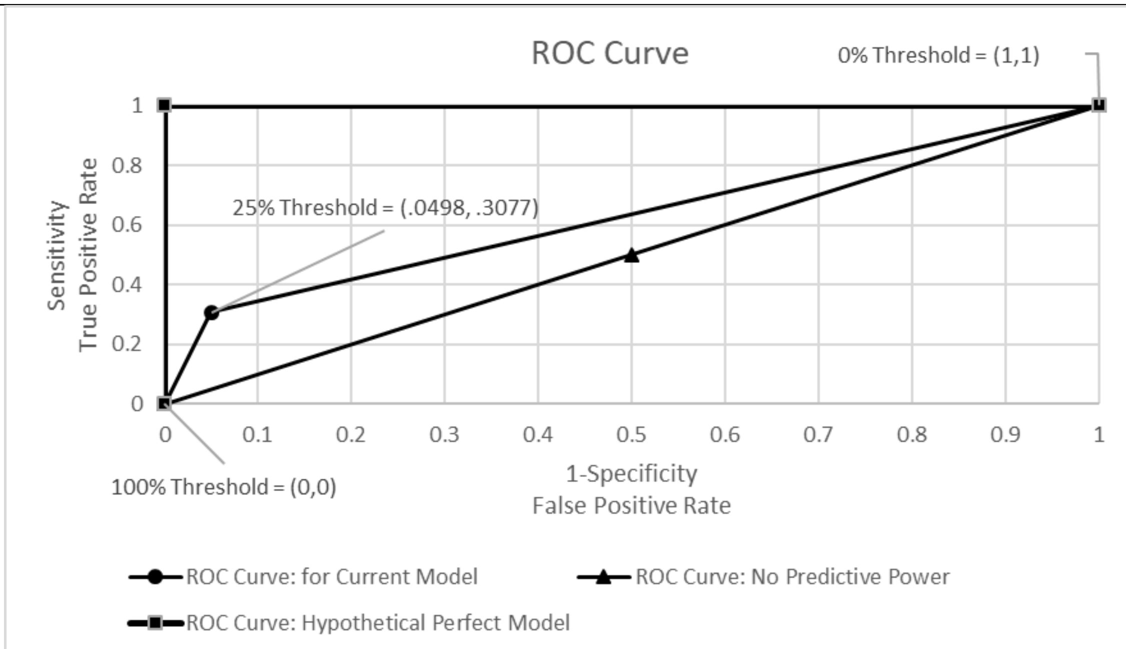
Appropriate b/c it maps to a range between 0&1, which is similar to a probability.

Part b: 0.5 point

$$Sensitivity = \frac{\text{True Positives}}{\text{Total Positive}} = \frac{72}{72+162} = 30.77\%$$

$$Specificity = \frac{\text{True Negatives}}{\text{Total Negatives}} = \frac{1203}{1203+63} = 95.02\%$$

Part c: 1.5 points



Part d: 0.25 point

Sample 1

The more severe the claims, the lower the discrimination threshold since the benefit of identifying fraudulent claims will outweigh the cost of investigating false negatives.

Sample 2

With high severity claims, we should lower the threshold so that more claims are predicted to be fraudulent and less fraudulent claims go undetected. Because the claims are more severe, the cost of investigating additional claims will be outweighed by the cost if those claims go undetected.

EXAMINER'S REPORT

Candidates were expected to know the appropriate link function to use for a generalized linear model (GLM) with a binary target variable, how to construct the receiver operating characteristic (ROC) curve and to understand the components within the ROC curve, and how the severity of claims impact the selection of the model threshold.

Part a

Candidates were expected to know which link function to use for a GLM that has a binary target variable.

Candidates did not receive full credit if they gave an incorrect link function or didn't explain why the link function would be appropriate for a binary target variable.

Common mistakes included:

- Mixing up the logit function and the logistic function
- Giving an incorrect link function such as:
  - Log
  - Binomial

- Negative Binomial
- Indicating an incorrect range of the linear predictor
- Saying the output of μ is either 0 or 1, without indicating that it is a range between 0 and 1.

Part b

Candidates were expected to know how to calculate the sensitivity and specificity from the confusion matrix.

A common mistake was using the incorrect denominator.

- For sensitivity, the most common incorrect denominator was adding the true positives and false positives
- For specificity, the most common incorrect denominator was adding the false negatives and true negatives.

Part c

Candidates were expected to plot the ROC curve with the 0%, 25%, and 100% discrimination thresholds, as well as the model with no predictive power and a hypothetical perfect model.

Common mistakes included:

- Mixing up the discrimination thresholds of 0% and 100%
- Incorrectly labeling the x-axis and y axis
    - Labeling the x-axis as sensitivity and the y-axis as 1 –specificity
    - Labeling the x-axis as specificity or false negative rate
    - Labeling the y-axis as true negative rate
- Incorrectly labeling or not labeling the discrimination threshold of 25%
- Not plotting the ROC curve for the current model, but instead just plotting the 25% threshold
- Incorrectly plotting the ROC curve or just plotting a point of (0, 1) for the hypothetical perfect model without drawing the ROC curve
- Incorrectly plotting the ROC for the model with no predictive power.

Part d

Candidates were expected to describe how the severity of claims will impact the selection of the model threshold.

Common mistakes included:

- Indicating that the severity of the model would not impact the selection
- Indicating that the threshold would increase if the severity increased
- Incorrectly describing how the sensitivity and false positive rate would be impacted by the severity
- Stating a preference to accept more false positive when severity was high, but did not explain how the threshold selection itself would be impacted by this preference.

QUESTION 6	
TOTAL POINT VALUE: 2	LEARNING OBJECTIVE(S): A3

SAMPLE ANSWERS

Part a: 0.75 point

Sample 1
Sprinkled Property, AOI 200K, Class 2 Fitted Frequency
= $e^{-8.4607+0.2714+\ 0.7228+\ 0.4311\ln(200000)-0.0960(1)\ln(200000)}$
$= 0.034176$

Non Sprinklered Property, AOI 200K, Class 2 Fitted Frequency
= $e^{-8.4607+0.2714+0.4311*\ln(200000)}$
$= 0.053543$

Ratio = 0. 034176/0.053543 = 0.638286

Sample 2
μ for sprinklered /μ for non-sprinklered
= $e^{.7228\ -.096\ln(200000)}$ (other terms cancel out)
= 0.6383

Part b: 0.75 point

Sample 1
New Intercept = -8.4607 + 0.4311ln(200,000) = -3.1987

Sample 2
Centered at base level of 200,000
- ➔ For 200,000 AOI, no coefficient

For non-sprinklered:
-8.4607 + 0.2714 + 0.4311ln(200,000) = -2.927 = b0 + 0.2714
New intercept = -3.1987

Part c: 0.5 point

Transformed intercept being indicated predicted target at base case
- Intercept represents all variable at their base levels -> easier to interpret
- The intercept term reflects average frequency at base levels, which is intuitive
- The intercept term represents your (untransformed) base rate

Sign of interacted variable
- Avoids non-intuitive interaction terms, such as a negative coefficient for low AOI non-sprinklered properties when base is not the center

- When variable is not centered, sometimes a coefficient may have the opposite sign than expected, this is especially true when an interaction term is present, so the coefficients are more intuitive to understand when centering variables
- When terms are not centered, you can have unintuitive results. E.g. the sprinkler coefficient is positive which can appear to indicate a higher frequency for sprinklered building

EXAMINER'S REPORT

Candidates were expected to know the components of a GLM formula and be able to calculate the output of the model based on the information provided. Also, they were expected to understand the transformation of variables and its impact on GLM output.

Part a

Candidates were expected be able to calculate the predicted frequency based on the output of the GLM model.

Common mistakes included:

- Providing the calculation for only one of Non-Sprinklered or Sprinklered
- Failing to recognize the log link function
- Error on the interaction term while calculating the frequency for Sprinklered.

Part b

Candidates were expected to understand how transformation of a continuous variable (Centering) and its impact on GLM output.

Common mistakes included:

- Not recognizing the new intercept as function of coefficient for ln(AOI)
- Including the interaction term in the adjustment
- Not applying additive adjustment to the original intercept.

Part c

Candidates were expected to provide advantages of transforming (centering) continuous variables while building a GLM.

Common mistakes included:

- Simply stating "easy to explain", "intuitive", "easy to calculate" without further detail
- Stating "p-value reduction, narrower confidence intervals, standard error reduction, and increased variable significance due to large exposure concentration" without explicit specification of the variable (Note that coefficients and their significance around "ln(AOI)" and "ln(AOI) and sprinklered" interaction do not change after the centering of AOI– refer to the GLM output on pages 56 and 57 of Goldburd et al. Credit was provided if the candidate has explicitly specified this rationale around the coefficient of "Sprinklered".)

- Stating the reduction of a variable in the model or degree of freedom as an advantage – transforming a continuous variable does not reduce the number of variables used in the GLM model, therefore, does not reduce the degree of freedom.

QUESTION 7	
TOTAL POINT VALUE: 7.5	**LEARNING OBJECTIVE(S): B1-B3, B6-B7, C3**
SAMPLE ANSWERS	
Part a: 2.25 points	

Sample 1

E[X] = $\frac{\beta}{\alpha-1}$

E[X;x] = $\frac{\beta}{\alpha-1}$ [1-$\left(\frac{\beta}{x+\beta}\right)^{\alpha-1}$]

LER = $\frac{E[X;100k]}{E[X]}$ = 1-$\left(\frac{22800}{122800}\right)^{0.3}$ $= 0.3966$

Expected limited loss 2016-18

= 1,064,000 x 0.3966 $\left(\frac{1}{1.286*1.045^2}+\frac{1}{1.094*1.045^3}+\frac{1}{1.052*1.045^4}\right)$

= 974,860

Sample 2

Limited Loss % = $\frac{E[X;L]}{E[X]}$ = $\frac{\frac{\beta}{\alpha-1}}{\frac{\beta}{\alpha-1}\left[1-\left(\frac{\beta}{x+\beta}\right)^{\alpha-1}\right]}$ = 1-$\left(\frac{22800}{122800}\right)^{0.3}$ $= 0.3965$

Annual limited expected loss = 1064 (.3965) = 421.9

-use 3 years of data lagged 1 year (use 16, 17, 18)

-average accident date is 7/1/2020, for 2018 this is 2 years of trend

	A	B	C	AxBxC
PY	Prospective Lim Loss	Detrend	De-develop	E[Lim Loss at Reported]
18	421.9	1.045^{-2}	1/1.286	300.4
17	421.9	1.045^{-3}	1/1.094	337.9
16	421.9	1.045^{-4}	1/1.052	336.3
				∑ = 974.6k

Alternate Solution:
Experience period: PY 2016-2018
*Assume "E(loss) before mod" refers to limited losses before experience mod

PY	E(Ult loss)	
2016	848,127	detrend 7/1/18-7/1/2020
17	852,267	undeveloping loss
18	757,649	= 1,064,000/$(1.045)^2$/1.286
Total	2,458,042	

Part b: 1.5 points

Sample 1
Limited reported losses = 392,457 – 128,305 + 1,013,863 – 525,626 – 152,860 + 459,798 – 275,865 + 400,000
= 1,183,462

Exp Mod = $1+z\left(\frac{A}{E}-1\right)$
= $1+0.4\left(\frac{1{,}183{,}462}{975{,}763}-1\right)$
= 1.085

Unlimited Expected Loss = 1,064,000 * 1.085 = 1,154,653

Sample 2

PY	Total Capped Rpt Loss
2016	392,457 – 128,305 + 100,000 = 364,152
2017	1,013,863 – 525,626 – 152,860 + 100,000*2 = 535,377
2018	459,798 – 275,865 + 100,000 = 283,933
Total	= 1,183,462 = expected gnd up reported loss capped at 100k

Z = $\frac{E}{E+K}$
ZE + ZK = E
$\frac{E\,(1-Z)}{Z}$=K
Z = 0.4
E = 974,644 (from a)
-> K = 1,461,966

Mod = $\frac{A+K}{E+K}$ = $\frac{1{,}183{,}462+1{,}461{,}966}{974{,}644+1{,}461{,}966}$
Mod = 1.0857

Total modified gnd up ult loss
=1.0857 * 1,064,000
=1,155,184

Alternate Solution
Actual reported loss limited to $100,000 per claim

Year	Actual Loss Limited	
2018	459,798 - (275,865 - 100,000)	283,933
2017	1,013,863 – (525,626 - 100,000) – (152,860 – 100,000)	535,377
2016	392,457 – (128,305 - 100,000)	364,152
	Total=	1,183,462

Mod = (Actual/Expected) *z + (1.0)*(1-z)
= (1,183,462/2,457,841)(0.40) + (1-0.40)
=0.7926

Modified groundup limited ult loss = (1,064,000)(0.7926)
=843,539

E[X; 100,000] = $\frac{22{,}800}{1.3-1}\left(1-\left(\frac{22{,}800}{100{,}000+22{,}800}\right)^{1.3-1}\right)$ = 30,139.95
E[X] = $\frac{22{,}800}{1.3-1}$ = 76,000
$\frac{30{,}139.95}{76{,}000}$ = 0.397

Modified ground up unlimited expected loss
= $\frac{843{,}539}{0.397}$ = 2,124,783

Part c: 0.25 point

Sample Responses

- This approach would give equal weight to all years of experience while the method we used in this exercise gives more weight to older years' experience

- By trending historical loss and keeping the expected at ultimate level, we're assigning equal weights to the expected loss. From part a, the detrended, un-developed losses do not have equal weights therefore the mod will be different between the methods.

- Using reported loss puts more weight on older years

Part d: 0.5 point

Sample 1

GCP = $\frac{E[L]*\ LAE\%}{1-V}$ = $\frac{1.064M*1.085*1.075}{1-0.035-0.17}$ = \$1.561M

Sample 2

GCP = (e+E)T

e = 0.075 * 1,154,563 = 86,592

T = $\frac{1}{1-.035-.17}$ = 1.258

GCP = (86,592 + 1,154,563) * 1.258 = \$1,561,373

Part e: 1.5 points

Sample 1:

ELG = 28 based on E[X] = 1,155,078

R_g = 2M/1,155,078 ≈ 1.75

R_h = 4M/1,155,078 ≈ 3.50

$E[X\}_r$ = 1,155,078 * (0.1131 – 0.0148)

= 113,544

Sample 2:

E[Loss] = 1,155,070

ELG = 28

R_{high} = 4,000,000/1,155,070 = 3.46

R_{low} = 2,000,000/1,155,070 = 1.73

$\phi_{3.46}$ = .0148

$\phi_{1.73}$ = .1131

so loss in layer = (.1131 - .0148) * 1,155,070

= 113,543.38

Part f: 1 point

Sample 1:

$$\frac{1{,}155{,}078 - 113{,}544 + 1{,}155{,}078 * (0.075) + 200{,}000}{1 - .035 - .017}$$

= 1,670,648

Sample 2:

P= 200k
Diff = 200k-113,546 =86,454
GCP from d. = 1,561,827
Primary insurer must charge:

$$1{,}561{,}927 + \frac{86{,}454}{1 - .035 - .017}$$

= 1,670,674

Sample 3:

$$\frac{(1{,}154{,}550) * (1 + 7.5\%) - 113{,}492 \ + \ 200{,}000}{1 - 3.5\% - 17\%}$$

= 1,669,999

Part g: 0.5 point

Sample 1:
The assumption of a Pareto distribution may not be accurate which will have a large impact on the tail of the distribution. This could make the insurance charges under or overstated making for an inequitable premium.

Sample 2:
The calculation in part f doesn't account for the primary insurer transferring a large portion of uncertain risk to the reinsurer. To account for this, the primary insurer could lower its profit or risk load.

EXAMINER'S REPORT

Candidates were expected to understand the actuarial principles and concepts underlying the development of experience rating plans. They were expected to demonstrate this knowledge by calculating the experience modification factor for a policy and the subsequent modified premium. Many candidates were unclear as to how the loss cap impacts the experience mod and the prospective premium.

Candidates were then expected to use a Table M lookup to determine the ceded loss and needed premium under a reinsurance treaty, and to comment on the impact of actuarial assumptions on these values.

Candidates should note that parts a through d of this question were very similar to steps 2, 3 and 4 of the Exam 8 Syllabus Case Study.

Part a

Candidates were expected to be able to bring prospective expected losses to the levels in the loss experience period on a limited basis in order to facilitate the calculation of an experience mod. Candidates needed to determine the correct trend period, calculate the appropriate loss elimination ratio (i.e. E[X; x] / E[X]) using the shifted pareto distribution, and apply the appropriate limited loss development factors.

Answers that assumed $1,064,000 was a *limited* ultimate expected loss were also accepted if the candidate calculated the appropriate ILF to use in part b.

Common mistakes included:

- Not calculating a loss elimination ratio
- Using reported losses to calculate a loss elimination ratio
- Applying an excess ratio instead of a loss elimination ratio
- Using F(100k) as a loss elimination ratio
- Using the limited expected severity as the ultimate loss
- Detrending the $100k loss cap to the experience period
- Adjusting the reported losses (e.g. trending and developing to the prospective policy period)
- Using the wrong trend period
- Assuming the given loss development factors were age-to-age factors
- Using 4% trend instead of 4.5%.

Part b

Candidates were expected to apply the individual loss cap to reported losses, select the appropriate policy years, calculate the experience modification using the credibility factor, and calculate the final unlimited expected loss.

Answers that assumed $1,064,000 was a limited ultimate expected loss were also accepted, but required the candidate to also calculate the appropriate increased limits factor using the shifted pareto distribution.

Candidates who skipped the mod calculation completely and calculated a credibility-weighted answer using unlimited expected and reported losses received very little credit.

Common mistakes included:

- Using unlimited reported and/or expected values to calculate the mod
- Trending or developing reported losses
- Dividing reported losses by the prospective expected loss
- Forgetting to calculate a final expected loss after calculating the mod.

Part c

Candidates were expected to describe a reason why trending and developing reported losses to the cost level of the prospective policy period may not produce an identical experience mod to the method calculated in part b.

Many candidates stated that the experience mod would differ due to the interaction of the loss trend and the loss cap. This answer was not accepted because the interaction is also contemplated when detrending and undeveloping losses to the experience period.

Common mistakes included:

- Stating that the methods would differ due to the loss cap
- Stating that projecting reported losses would be a less credible method
- Stating that reported losses may differ from the assumed shifted Pareto distribution.

Part d

Candidates were expected to calculate the final guaranteed cost premium using the experience-modified unlimited expected losses, the LAE %, and the other variable expense components.

Common mistakes included:

- Ignoring the effect of the experience mod when calculating the expected loss
- Assuming the policy was a large deductible or retrospectively rated policy
- Using limited losses, or otherwise assuming a prospective policy limit of $100k
- Using reported losses as expected loss
- Using the wrong LAE load (7% vs 7.5%).

Part e

Candidates were expected to use the modified ground up unlimited expected loss as calculated in part b and the parameters of the reinsurance treaty to determine the ELG, entry ratios, and table M charges. Using the difference between the table M charges and the modified ground up unlimited expected loss, they then calculated the loss expected to be ceded to the reinsurer. Any answer from part b., regardless of rounding or calculation errors, could be pulled forward to part e. and potentially receive full credit.

Candidates received small deductions when using the expected ultimate loss before modification, even if they stated that as their assumption. Candidates were not penalized for assuming the closest ELG or table M charge when their calculated loss or entry ratio fell outside the given tables.

Common mistakes included:

- Using the unmodified loss
- Using modified losses to determine the ELG, but not the entry ratios or vice versa.
- Calculating insurance savings instead of insurance charge
- Attempting to interpolate values instead of rounding to the nearest entry ratio.

Part f

Candidates were expected to use the expected ceded loss calculated in part e to calculate the new premium the primary insurer must charge. Most successful candidates calculated the retained losses, LAE, and reinsurer premium as shown in Sample 1. Another common response was to calculate the additional premium needed due to reinsurance and add that number to the guaranteed cost premium from part d as shown in Sample 2. Candidates could calculate a mathematically equivalent answer without directly calculating retained losses, which is shown in Sample 3. All correct solutions maintained the 0% profit provision, treated the reinsurance as a fixed expense, and accounted for variable expenses. Any answer from part b., d. and/or e., regardless of rounding or calculation errors, could be pulled forward to part f. and potentially receive full credit.

Common mistakes included:

- Calculating a profit provision for the reinsurer, and then calculating a new premium for the primary insurer using that provision.
- Failing to correctly adjust for variable expenses.
- Calculating only the additional premium the primary insurer should charge for the reinsurance treaty, but not the total premium.

Part g

Candidates were expected to reference one of several assumptions the actuary made in this problem and describe how that assumption would affect the premium if incorrect. The most common correct answers referenced the shifted Pareto claim severity or the assumption that the reinsurer and primary insurer should maintain the same underwriting profit despite the transfer of a risky layer to the reinsurer.

Candidates generally did not receive full credit for creating an assumption not mentioned in the problem and then stating that it wasn't met. For example, a candidate might state that the given Table M had not been adjusted for the loss limitations, and then state that this was inappropriate. This question was challenging for most candidates.

Fisher et. al. (p.96) discusses the potential mismatch of the distributions underlying the expected loss and aggregate charges as a source of error. Clark (p. 42) discusses how the parameter variance of the aggregate distribution can lead to errors. A discussion of either of these responses would have been acceptable, full credit answers.

Common mistakes included:

- Stating that a 0% profit provision was inherently inequitable, which is not true due to investment income.
- Stating that it was unreasonable for the reinsurance premium to be treated as fixed.

QUESTION 8	
TOTAL POINT VALUE: 1.5	LEARNING OBJECTIVE(S): B1
SAMPLE ANSWERS	

Sample 1

ILF(500K) = 156,250/58,750 = 2.6596
ILF(1M) = 218,750/58,750 = 3.7234

Expected losses = 65,000*(3.7234 - 2.6596) = 69,149

Premium = 69,146*1.2*1.015 / (1 - 0.15 - 0.025) = 102,089

Sample 2

Assume the profit load is proportional to loss and LAE

Basic prem = (65,000*1.2*1.015*1.025)/(1 - 0.15) = 95,470

ILF (1M) = 218,750/58,750 = 3.723
ILF(500K) = 156,250 / 58,750 = 2.660

95,470*(3.723 - 2.660) = 101,564

Sample 3

E[x; 500k+500k] - E[x; 500k] = 218,750 - 156,250 = 62,500
E[S] = E[N]*E[X] = 65,000 = E[N]*58,750
E[N] = 1.1064

Expected layer loss = 1.1064*62,500 = 69,149

Premium = [69,149(1.20)]*1.015 / (1 - 15% - 2.5%) = 102,089

Sample 4

Expected indemnity 500k x 500k =
65k * (218,750/58,750 – 156,250/58,750) = 69,149

Assume ALAE and ULAE are a percent of ground-up losses

ALAE = 20% * 65k * 218750/58750 = 48,404
ULAE = 1.5% * [65k * 218750/58750 * (1+20%)] = 4,356

Premium = (69,149 + 48,404 + 4,356) / (1 – 15% - 2.5%) = 147,769

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
Candidates were expected to calculate ILFs using the correct expected severity from the given table and use the other information to calculate the premium of the policy. As the question did not specify the profit as a percentage of premium, candidates assuming the profit as a percentage of loss also received credit. Candidates also received credit by assuming the ALAE and ULAE were a percent of ground-up losses. Common mistakes included: • Mistaking layer severity as layer loss cost • Calculating ULAE as percentage of loss instead of loss and ALAE.

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 9	
TOTAL POINT VALUE: 1	**LEARNING OBJECTIVE(S): B3**
SAMPLE ANSWERS	

Sample Responses for Criteria 1

- High VHM means there is significant variance between risks due to risk difference, this cause credibility of experience to increase, making experience rating useful.
  High EVPV means there is a lot of volatility in loss experience. This causes credibility of experience to decrease making experience rating not useful.
  Thus experience rating is most useful for class B since it has high VHM and low EVPV,
  Least useful for class C since it has low VHM and high EVPV.

- Experience rating is used to further refine the classification plan beyond existing classes. It works best when there is high variance within each class and low process risk, so it would work best for class B because it has high VHM and low EVPV.
  Experience rating is least successful when the class itself is fairly homogeneous already and the loss experience is very volatile, because this it would be least successful for class C with low VHM and high EVPV.

- Class B will have the most benefit from experience rating
  High VHM and low EVPV means the variation is mostly driven by differences in the actual loss experience, and little is due to the randomness of the insurance business.
  Higher proportion of variance explained by difference in loss costs.
  Class C will have the least benefit from experience rating.
  High EVPV, low VHM means that the class is fairly homogeneous in terms of experience but the variance exists due to randomness of data.
  Lower proportion of variance can be explained by difference in loss costs.

Sample Response for Criteria 2

- Credibility $Z = n / (n + k)$, $k = EVPV/VHM$
  High EVPV and Low VHM = Low Credibility
  So Class C is the least useful because it has high EVPV + low VHM, thus low credibility for experience rating.
  Class B is the most useful because it has high credibility for similar reason.

EXAMINER'S REPORT

Candidates were expected to demonstrate an understanding of when experience rating may or may not be useful in practice. Related concepts include the credibility formula, the variance of hypothetical means, and expected process variance.

Candidates were either expected to
a) select the appropriate classes (most & least useful)
b) provide either:
1) a sentence or two exhibiting some qualitative knowledge of VHM and EPV
2) produce the credibility formula and explain the quantitative effects of VHM and EPV.

Common mistakes included:

- Selecting the correct classes, but not providing enough explanation, or an explanation that simply restated the terms provided in the question (High VHM, Low EPV)
- Selecting A/D classes instead of B/C
- Switching VHM and EPV when writing the k formula, but then supporting the correct answers verbally based on correct k: credit was dependent on additional details
- Failing to acknowledge a link between the credibility formula (Z) and the usefulness of experience rating in a rating plan
- Failing to select the most/least useful after identifying the most credible class.

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 10	
TOTAL POINT VALUE: 1.5	**LEARNING OBJECTIVE(S): B4**
SAMPLE ANSWERS	
Part a: 0.5 point	

Sample 1
Implement to charge more equitable premiums and avoid adverse selection. Experience mod is correcting for risk differences

Sample 2
Standard loss ratio is increasing, which indicates credibility to the experience is not high enough. Without the experience modification factor, the manual loss ratio would have a steeper increasing trend than the standard loss ratio. So the proposed plan should be implemented

Sample 3:
Standard LR=loss/standard prem=loss/(manual prem * mod) = manual LR/Mod
Without proposed mod, loss ratios would have more dispersion (ex quintile 1 manual LR=0.8*.4 = 32%) The mod appears to be identifying risk differences and somewhat correcting them

Sample 4:
Yes, the standard LR of proposal is flatter than the E-mod line, indicating the model is doing a good job of correcting manual LRs for differences in risks.

Sample 5:
As exp mod increases, the std LR of proposed plan still has an upward trend as opposed to flat. However there is no experience rating currently implemented so the manual LR curve would be even steeper, so there is currently even less individual risk equity. I would implement the proposal to improve risk equity.

Sample 6:
Yes because the new mod reduces the variance of the SLR relative to the manual loss ratio

Sample 7:
From the graph we see that the plan does not perfectly produce a standard LR that's flat across quintiles, but it is better than not having mod at all. E.g. for quintile 1, if the mod is not applied, the premium would be higher and the SLR would be even lower, which is not desirable. Thus it should be implemented.

Sample 8:
Yes, the graph shows there can be better segmentation of risks from low to high and the experience rating will allow the insurer to achieve same level of profitability across all risks

Sample 9:
I would implement the new mod because the standard loss ratios are flatter than the mod factor which suggests the mod factor does a somewhat decent job at correcting for differences

between risks. Without the experience plan, worse than average risks will be even more underpriced and the company could be adversely selected against

Sample 10:

Quintile	SLR	Mod	MLR=SLR*Mod
1	0.79	0.42	0.33
2	0.81	0.62	0.50
3	1.00	1.00	1.00
4	1.03	1.20	1.24
5	1.28	1.65	2.11

So since MLR have high positive trend, the model is good at identifying risk difference. The model partially corrects for risk difference because in the SLR there is small positive trend. Model doesn't give enough cred. It is better than the current model, but can be improved. Should be implemented.

Part b: 0.5 point

Sample 1:
Not enough credibility is given to actual experience because of increasing standard loss ratios, so give more credibility to actual experience

Sample 2:
Give more weight to actual experience, this will reduce premium for low mods and increase for high mods. The final product should be standard LR that do not have a trend

Sample 3:
The results can be improved by increasing the credibility. Reducing K in Z = E/(E+K) given to actual loss experience

Sample 4:
Experience mod results can be improved by increasing the mod in quintile 5 and decreasing in quintile 1 so that standard loss ratios are more consistent across quintiles

Sample 5:
Give high mod risk more of a debit and low mod risk more of a credit to avoid anti-selection

Part c: 0.5 point

Sample 1:
No, do not use. Current standard LR is flatter than proposed SLR. Also efficiency test shows same conclusion. Current<proposed. Current is better at accounting for experience.

Sample 2:
Do not use proposed plan. Standard loss ratios should be close to constant (supports current) and prefer a lower efficiency statistic (supports current)

Sample 3:

Proposed plan has higher efficiency test statistics and also the standard LR has an upward trend. The current plan works better
EXAMINER'S REPORT
Candidates were expected to be able to be able to compare the Manual LR to the Standard LR.
Part a
Candidates were expected to be able to identify/calculate the Manual LR from the graph provided and compare it to the Standard LR. Two of the points must have been made in order to receive full credit. A common mistake was stating that the plan should not be implemented since the Standard LR was not flat.
Part b
Candidates were expected to be able to identity that not enough credibility was assigned to the loss data since there as an increasing trend in the Standard LR. Full credit was awarded for one explanation of how to improve the model and for an explanation on why the model needs to be improved, or an additional reason or if both reasons on how to improve the model were provided. If candidate wrote about the credibility in Part A and did not mention everything in Part B, the responses in Part A was considered for Part B since this was a common mistake. Simply stating that the credibility needed to be adjusted was not an acceptable response.
Part c
Candidates were expected to be able to compare quintile charts as well as efficiency test statistics.

QUESTION 11	
TOTAL POINT VALUE: 3.75	**LEARNING OBJECTIVE(S):** 3, 4
SAMPLE ANSWERS	
Part a: 0.5 point	
Sample 1 X = Z*(A/E) + (1-Z) = 0.75*(37,100/32,000) + (1 - 0.25) = 1.1195 *Sample 2* Mod = X = 1 + (.75)*(37,100 – 32,000)/32,000 = 1.1195 *Sample 3* [37,100(0.75) + 32,000(0.25)] / 32,000 = 1.12	
Part b: 0.5 point	
Sample 1 Cap claims at 4000. This includes all of the smaller claims but prevents larger claims from unduly affecting the mod. [Various capping thresholds and claim removal proposals were accepted] *Sample 2* Standard LR=Manual LR/Mod Standard LR for risk 3 = .96 / 1.1195 = .8575 This is not close to other risks' standard loss ratio. Mod should be lower (closer to 0.96) so it should be given less credibility (more weight on expected experience and less on actual experience).	
Part c: 2.75 points	
Sample 1 (claim capping) Without update: Y = 0.96/1.1195 = 0.8575 Efficiency Stat: Var(Manual LR)/Var(Std LR) Avg Manual LR = (0.6 + 0.8 +… + 1.15)/5 = 0.922 Var(Manual LR) = ((0.6-0.922)^2 + (0.8-0.922)^2 + … + (1.15-0.922)^2)/(5-1) = 0.05092 (using sample variance) Avg Std LR = (1+1.01+0.8575+1+1.01)/5 = 0.9755 Var(Std LR) = ((1-0.9755)^2 + (1.01-0.9755)^2+ … + (1.01-0.9755)^2)/(5-1) = 0.004376 Eff Stat_w/o update = 0.004376/0.05092 = 0.0859 With update: New Mod = (0.75*32,100 + 0.25*32,000)/32,000 = 1.0023 (actual losses limited to 5k) Y = 0.96/1.0023 = 0.9578 Var(Manual LR) = 0.05092 (same as without update) Avg Std LR = (1+1.01+0.9578+1+1.01)/5 = 0.99556	

Var(Std LR) = ((1-0.99556)^2 + (1.01-0.99556)^2+ … + (1.01-0.99556)^2)/(5-1) = 0.0004706
Eff Stat_w/o update = 0.0004706/0.05092 = 0.009242
Since 0.009242 < 0.0859, loss cap improves plan.

Sample 2 (decrease credibility)
New Mod = [0.25(37,100) + 0.75(32,000)]/32,000 = 1.04
Y = 32,000 / (33,333 * 1.04) = 92.3%

Old		New	
Manual	Standard	Manual	Standard
0.60	1.00	0.60	1.00
0.80	1.01	0.80	1.01
0.96	**0.86**	0.96	**0.92**
1.10	1.00	1.10	1.00
1.15	1.01	1.15	1.01

Eff test_Old = 0.00384 / 0.040736 = 0.0715
Eff test_New = 0.001176 / 0.040736 = 0.029 → lower test statistic implies improvement

EXAMINER’S REPORT

Candidates were expected to demonstrate knowledge of experience rating plans, including how to calculate experience rating modification factor, manual and standard loss ratios, evaluation of the effectiveness of an experience rating plan, and how to modify an experience rating plan to improve its effectiveness.

Part a

Candidates were expected to provide the formula used to calculate the modification factor for Risk 3, and to provide the final modification factor in numeric form.

A common mistake included:

- Making a simple calculation error

Part b

Candidates were expected to provide a reasonable recommendation to improve the effectiveness of experience rating for Risk 3, and briefly justify the recommendation.

Common mistakes included:

- Providing a correct recommendation but not justifying it
- Providing a recommendation that did not improve the effectiveness of experience rating for Risk 3.

Part c
Candidates were expected to calculate a new mod and standard loss ratio (X & Y) based on the recommendation in part b, perform an efficiency test on both the original and recommended experience rating plans, and explain the result of the test. Full credit was given for various forms of variance calculation (e.g., sample or population variance or standard deviation). Partial credit was awarded for a qualitative response based on the quintiles test. Candidates should note, though, that Fisher et. al. (p. 10) specifies that the quintiles test is a qualitative test (observing general trends) , and this question specifically asked for a quantitative demonstration (calculating and comparing two quantities to support a conclusion). Common mistakes included: • Calculating a test statistic for the recommended plan but failing to calculate one for the original plan • Not drawing a conclusion on the results of the test • Multiplying SLR = MLR * Mod rather than dividing SLR = MLR / Mod.

QUESTION 12	
TOTAL POINT VALUE: 2.0	**LEARNING OBJECTIVE(S): B2**
SAMPLE ANSWERS	
Part a: 0.5 point	

Sample 1
Subset B has a significantly different loss distribution than subset A. When calculating insurance charges, subset A's would be too high and subset B's would be too low.

Sample 2
A and B have different loss distributions, even though they have the same expected loss. B is more volatile than A ➔ it should have a higher insurance charge than A for the same entry ratio ➔ Combining A and B to create 1 table M will underprice B and overprice A.

Sample 3
The severity distribution of A and B are different and the variance of A and B is different. The insurance charge is dependent the severity distribution and variance. If A and B are combined, the charges and savings from the Table M will be incorrect.

Sample 4
The policies in subset A, compared to subset B, have a smaller variance. Table M's are selected based on the variance of losses for an insured. Using a single Table M for the two subsets is not appropriate since they have different levels of volatility.

Sample 5
It seems that subset B has more variation in aggregate losses than subset A, indicating that separate table M's should be calculated for each subset because of the different aggregate loss distributions.

Part b: 1 point

Sample 1
i. $(80k - 10k) \times 0.1 + (40k - 10k) \times 0.3 = 16k$
ii. $(80k - \ 40k) \times 0.1 = 4k$
iii. 0

Sample 2
i. $\emptyset(10k) = \frac{1\times(80k-10k)+(40k-10k)\times 3}{10} = 16{,}000$
in % → 16k/26k = 61.5%
ii. $\emptyset(40k) = \frac{(80k-\ 40k)}{10} = 4{,}000$
in % → 4k/26k = 15.4%
iii. 0
in % → 0%

Sample 3

$Total\ Area = 10(6) + 40(3) + 80 = 260$

i. $\emptyset(10) = \frac{(80-10)+(40-10)x3}{260} = .615$

ii. $\emptyset(40) = \frac{(80-40)}{260} = .154$

iii. 0, because there are no losses above $80k

Sample 4

$$r\ @\ 10{,}000 = \frac{10}{26} = .385$$

$$r\ @\ 40{,}000 = \frac{40}{26} = 1.538$$

$$r\ @\ 80{,}000 = \frac{80}{26} = 3.077$$

$$\emptyset(.385) = (1.538 - .385) \times .3 + (3.077 - .385).1 = .6151$$

$$\emptyset(1.538) = (3.077 - .385) \times .1 = .1539$$

$$\emptyset(3.077) = 0$$

Part c: 0.5 point

Sample 1

1. Need to consider if the different line of business has the same expected risk size
2. Should consider if the other line is more or less risky (different variance)

Sample 2

3. The insurance charge depends on the variance and shape of the severity distribution.
4. Consider whether the other line of business is subject to per-occurrence limits

Sample 3

5. The other line of business should have a similar expected loss to the Table M used, around $26,000
6. It is probably unwise to use a Table M built off of only 10 observations, especially for another line of business.

-

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of Table M for insurance rating including the calculation of Table M values and the assumptions underlying its use.

Part a

Candidates were expected to note the difference in loss distribution between subset A and B, and explain why this difference in loss distribution made the use of the current table M for both subsets inappropriate or inaccurate. Candidates that noted variation between the two subsets but did not explain the implications of those differences received partial credit.

Common mistakes included:

- Failing to explain why a difference in claim distribution makes the current Table M inappropriate
- Suggesting that the risks in the two subsets were of different sizes (each risk had exactly one claim, so variation was the result of claim severity, not risk size)
- Giving answers that were too vague (e.g. "subsets look different" or "there is too much variance")
- Stating that overall variance of the portfolio makes Table M unreliable

Part b

Candidates were expected to accurately calculate insurance charges. Insurance charges calculated as either dollar amounts or ratios received full credit. Tabular calculations as well as formulas were both acceptable as well.

Common mistakes included:

- Failing to divide by the number of risks (10) when calculating insurance charges in the form of expected aggregate excess losses
- Subtracting 10,000 rather than 40,000 from 80,000 in part 2
- Dividing the loss amounts by the claim sizes rather than expected aggregate excess loss amounts
- Using incorrect insurance charge formulas.

Part c

Candidates were expected to demonstrate knowledge of the important considerations in the application of Table M. Full credit was given to a variety of responses, including but not limited to:

- The other line must have similar risk sizes/expected loss, (or that entry ratios could be used to account for scale difference)
- The other line of business of must have similar severity or aggregate loss distribution/variance
- Noting that the Table M from part B may lack credibility due to limited data
- Noting that the other line of business should be subject to the same limit structure

Common mistakes included:

- Providing two responses that were deemed too similar (e.g. noting that claim variance should be similar for part 1, and claim distributions should be similar for part 2)
- Generic or vague responses that did not apply directly to the posed question
    - E.g. responses about general characteristics of Table M, such as $\phi(r)$ being a decreasing function
    - Suggesting that risks in the new line of business should be similar to part B, without an adequate explanation of the ways in which they must be similar
- Noting that Table M charges should be the same for both lines of business.

QUESTION 13	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): B2**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

Known Information

$E[S] = E[N](E[X;5000] - E[X;4000])$

$E[X;Y] = \frac{\beta}{\alpha-1}\left(1 - \left(\frac{\beta}{Y+\beta}\right)^{\alpha-1}\right) \quad \rightarrow \quad E[X;4000] = \frac{\beta}{\alpha-1}\left(1 - \left(\frac{\beta}{4000+\beta}\right)^{\alpha-1}\right)$= 1433.30

$E[X] = \dfrac{\beta}{\alpha - 1} = 3000 \quad \rightarrow \quad \beta = 3000(\alpha - 1)$

$e_x(x) = \dfrac{x+\beta}{\alpha - 1} \quad \rightarrow \quad e_x(6000) = \dfrac{6000+\beta}{\alpha - 1} = 15000$

Find β and α

$\dfrac{6000+\beta}{\alpha - 1} = 15000$

$6000 + \beta = 15000(\alpha - 1)$

$6000 + 3000(\alpha - 1) = 15000(\alpha - 1)$

$6000 = 12000(\alpha - 1)$

$\alpha = 1.5 \qquad \beta = 3000(\alpha - 1) = 1500$

Find the value of $E[X;5000]$

$E[X;5000] = \dfrac{\beta}{\alpha - 1}\left(1 - \left(\dfrac{\beta}{5000+\beta}\right)^{\alpha-1}\right) = \dfrac{1500}{1.5-1}\left(1 - \left(\dfrac{1500}{5000+1500}\right)^{1.5-1}\right) = 1558.85$

Find the value of $E[N]$

$E[N] = 0.5 \times 0 + 0.5 \times (0.7 \times 1 + 0.2 \times 3 + 0.1 \times 5) = 0.9$

Find the value of $E[S]$

$E[S] = E[N](E[X;5000] - E[X;4000]) = 0.9 \times (1558.85 - 1433.30) = 112.995$

Sample 2

Average excess claim size in layer 1000 xs 4000

$$= \frac{(E[X;5000] - E[X;4000])}{1 - F_x(4000)}$$

Since $E[X] = \frac{\beta}{\alpha-1}$, $\quad E[X;Y] = \frac{\beta}{\alpha-1}\left(1 - \left(\frac{\beta}{Y+\beta}\right)^{\alpha-1}\right)$, $\quad e_x(x) = \frac{x+\beta}{\alpha-1}$, $\quad F_x(x) = 1 - \left(\frac{\beta}{x+\beta}\right)^{\alpha}$

$\frac{\beta}{\alpha-1} = 3000$, $\frac{6000+\beta}{\alpha-1} = 15000 \qquad \alpha = 1.5 \;\; \beta = 1500$

$$\frac{(E[X;5000] - E[X;4000])}{1 - F_x(4000)} = \frac{(1558.85 - 1433.3)}{1 - 0.8576} = 881.5$$

Expected number of claims
$= 0.5 \times (1 \times 0.7 + 3 \times 0.2 + 5 \times 0.1) = 0.9$

Expected number of claims in layer
$= 0.9 \times \left(1 - F_x(4000)\right) = 0.1282$

Expected aggregate loss in layer
$= 881.5 \times 0.1282 = 113$

Sample 3

$$E[X] = \frac{\beta}{\alpha - 1}, \quad E[X;Y] = \frac{\beta}{\alpha - 1}\left(1 - \left(\frac{\beta}{Y + \beta}\right)^{\alpha-1}\right),$$
$$e_x(x) = \frac{x + \beta}{\alpha - 1}, \quad F_x(x) = 1 - \left(\frac{\beta}{x + \beta}\right)^{\alpha}$$

$\frac{\beta}{\alpha-1} = 3000$, $\frac{6000+\beta}{\alpha-1} = 15000 \qquad \alpha = 1.5 \;\; \beta = 1500$

Expected number of claims
$= 0.5 \times (1 \times 0.7 + 3 \times 0.2 + 5 \times 0.1) = 0.9$

Expected number of claims in layer
$= 0.9 \times \left(1 - F_x(4000)\right) = 0.1282$

Expected aggregate loss in layer

$$= \frac{(E[X;5000] - E[X;4000])}{1 - F_x(4000)} \times 0.9 \times \left(1 - F_x(4000)\right) = 0.9 \times (E[X;5000] - E[X;4000])$$
$$= 0.9 \times (1558.85 - 1433.3) = 112.995$$

Part b: 0.5 point

- The aggregate layer has fewer observations (loss experience) to fit, thus data is very volatile and thin, which makes it harder to estimate. And it can be difficult to smooth the

- layers between lower and higher areas to have a good smoothed transition when using the fitted curve.
- Excess losses tend to be sparse and may not be credible enough to give a consistent distribution fitting without some jump discontinuities.

EXAMINER'S REPORT

Candidates were expected to understand how to combine frequency and severity distributions into an aggregate loss distribution and to calculate the expected aggregate loss for a specified layer. Candidates were then expected to demonstrate an understanding of the realities of approximating an aggregate distribution with losses in an excess layer.

Part a

Candidates were expected to evaluate the α and β parameters of the severity distribution from the given information, before calculating the expected severity in the excess layer under consideration and combining with the unconditional expected claim count.

Common mistakes included:

- Incorrectly calculating the parameters α and β
- Using the conditional instead of unconditional expected claim count (i.e. forgetting to adjust for the 50% probability of 0 claims)
- Calculating the expected severity for the incorrect layer.

Part b

Candidates were expected to recognize the particularities of excess layers and to identify the small expected claim count as one of the drivers resulting in jump discontinuities in the cumulative distribution function.

Common mistakes included:

- Merely specifying that data is volatile, without reference to claim count
- Alluding to the required independence of frequency and severity distributions
- Neglecting to relate the small expected claim count to the resulting jump discontinuities

QUESTION 14	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): B5**
SAMPLE ANSWERS	
Part a: 1 point	

Sample 1

B = e – (c-1)*E[A] + cI
= 20,000 – (1.07 -1)*(825,000) + 1.07 (E[A] * (0.4 – (0.8+0.5-1)))
= 20,000 – 57,750 +88,275
= 50,525

Sample 2

$\psi(r_H)$ = 0.8 + 0.5 – 1 = 0.3
I = (0.4 – 0.3) * 825,000 = 82,500
B = e – (c-1)*E[A] + cI = 20,000 – (1.07-1)*825,000 + 1.07*82,500 = 50,525

Sample 3

Assume this is balance plan.
$\phi(r_H) - \phi(r_G)$ = (E+e-H/T)/cE
H = (B + cr_HE)*T
H/T = B + cr_HE
$\phi(r_H) - \phi(r_G)$ = (E+e-(B+cr_HE))/cE
B + cr_HE = E + e – cE($\phi(r_H) - \phi(r_G)$)
B = E + e – cE($\phi(r_H) - \phi(r_G)$) – cr_HE
= 825,000 + 20,000 – 1.07 * 825,000 * (0.8 – 0.4) – 1.07 * 0.5 * 825,000
= 845,000 – 353,100 – 441,375
= 50,525

Sample 4

Assume "expense" of 20k excludes LAE and refers to overhead expense.
B = e + cI
$\psi(r_H) = r - (1 - \phi(r_H))$ = 0.5 – (1 – 0.8) = 0.3
I = (0.4 – 0.3) * 825,000 = 82,500
B = 20,000 + 1.07*82,500 = 108,275

Sample 5

$\psi(r_G) = \phi(r_G) + r_G - 1$
0.4 = $\phi(r_G)$ + 1.21 -1
$\phi(r_G)$ = 0.19
I = ($\phi(r_G) - \psi(r_H)$) * E[A] = (0.19 – 0.8) * 825,000 = -503,250
B = e – (c-1)*E[A] + cI = 20,000 – (1.07-1)*825,000 + 1.07*(-503,250) = -576,228
This problem seems flawed: savings is higher @ the "H" which implies a negative net insurance charge. But solved based on information given in problem.

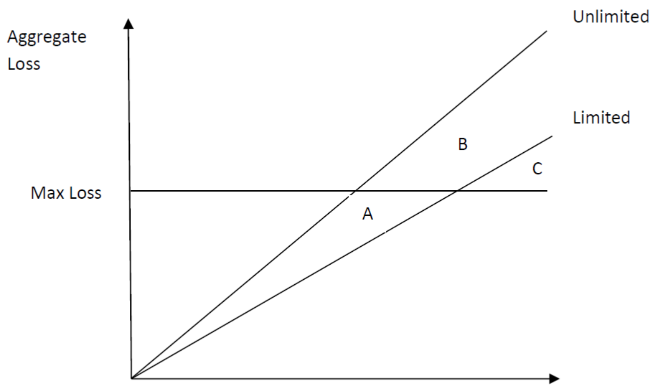
Sample 6 $\psi(r_G) = \phi(r_G) + r_G - 1$ $0.4 = \phi(r_G) + 1.21 - 1$ $\phi(r_G) = 0.19$ Assume there is a typo in the question and $\psi(r_H) = 0.08$, not 0.8. Otherwise, we get a large negative basic premium. B = e – (c-1)*E[A] + cI = 20,000 – 0.07 * 825,000 + 1.07 * (0.19 – 0.08) * 825,000 = 59,352.50
Part b: 1 point
Sample 1 • Some of the limited losses will also be capped by the maximum ratable loss -> Overlap between per-occurrence limit & aggregate limit is not removed in the actuary's method. • Because per-occurrence limit reduces the variance of the loss distribution -> charge for limited loss should be lower • Over estimate insurance by using unlimited table M values • Overall, the actuary will over price the risk. *Sample 2* • There is an overlap between the per occ limit and the agg limit set by the max premium (takes longer to reach max given limited losses), so it is not appropriate to load the full amount of $400,000 into the premium. • This will result in overcharging the insured. • The actuary should use a limited table M by using limited losses to per occ limit and add charge to per occ in excess of agg limit or a table L instead. *Sample 3* • It's not correct. Since the inclusion of per-occurrence limit will make the limited aggregate loss distribution less likely to reach the aggregate loss corresponding to max premium comparing to the unlimited aggregate loss distribution, thus the net insurance charge for the policy will be less. There's overlap of loss excess of per-occurrence limit and insurance charge of aggregate loss distribution. As a result, the actuary should not load the full 400,000. *Sample 4* • This is not appropriate. The presence of a per occurrence limit decreases the volatility in agg loss distribution in comparing a table M, thus $\phi(r_G)$ will decrease. There will be less loss in excess of agg limit of limited loss compared to unlimited loss. • Adding 400,000 excess per occurrence loss to basic premium in (a) will overestimated the basic premium, due to the overlap of maximum premium and per occurrence excess charge.

Sample 5

- The actuary needs to use a limited Table M or Table L to account for the per-occurrence limit being added. The insurance charge portion of the basic premiums needs to be modified to account for the per-occurrence limit (which will lead to less variance in aggregate losses and a different charge). The current table M charge will be too large since it's based on agg losses with no occ limit, so the insured would be charged for the occ limit twice if the actuary adds on expected losses exceeding the occ limit without adjusting the insurance charge portion of the basic premium.

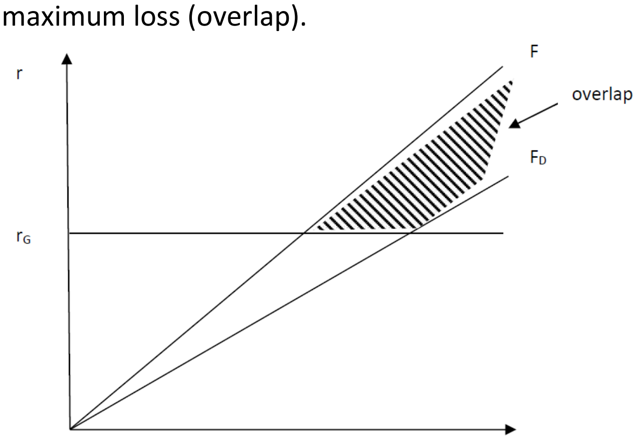
Sample 6

- This is inappropriate as by adding per occurrence limit it will reduce variance of losses & likelihood of hitting agg limit. This would result in a lower insurance charge as part of B. Without lowering insurance charge would be on overcharge as there would be an overlap in what insurance charge is and losses excess of occ. Need to lower insurance charge in addition to account losses excess occ.



Sample 7

- k*E[A] = 400,000
  Per occ limit = 200,000
  They can use limited table M, so
  $B = e - (c-1)*E[A] + c\,(\phi(r_G)^* - \psi(r_H)^*)*E[A_D] + ckE[A]$
- The method of this actuary is inappropriate because they will have a basic premium too high than correct b. The charge needs to diminish to be correct b in his approach because now he is double counting for losses that are above per occurrence limit and above maximum loss (overlap).



SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
Candidates were expected to understand the actuarial principles and concepts underlying the construction of a retrospective rating plan. Candidates were expected to understand how to calculate a basic premium, and how the interaction between a per occurrence limit and an aggregate limit impacts the basic premium in a retrospective rating plan.
Part a
Candidates were expected to know how to calculate the insurance charge (I) and basic premium (B) given the other parameters of a retrospective rating plan. $I = [\phi(r_G) - \psi(r_H)] * E(A)$ $B = e - (c-1)*E(A) + cI$ The symbol φ used in the exam for insurance charges confused some candidates because they were more familiar with the symbol ϕ for insurance charges. Consequently, some candidates interpreted the insurance charges given in the question as insurance savings (ψ). Those who did this and also stated a reason why the savings, entry ratios, or calculated charges didn't make sense received full credit if no other errors were made. Common mistakes included: • Using the wrong formula for I • Using the wrong formula for B (commonly adding (c-1)*E(A) rather than subtracting it).
Part b
Candidates were expected to fully describe the reason Table M cannot be used without adjustment in the presence of a per occurrence limit. Candidates were expected to explain the overlap that occurs between losses excess of the per occurrence limit and the aggregate limit. They were then expected to assess the impact of the overlap on the basic premium and conclude that the actuary's methodology overstates the basic premium. Some candidates discussed expenses and profit, rather than losses. Partial credit was awarded for valid expense and profit arguments. Common mistakes included: • Offering a solution of limited Table M, Table L or ICRLL, rather than explaining the problem that they solve • Drawing a Lee Diagram, but not identifying the correct area of overlap • Stating the actuary's method overestimated the basic premium without supporting reasons • Stating the actuary's method is a reasonable way to account for the excess portion of expected loss • Stating the actuary's method will underestimate the charge because there is an overlap with the aggregate limit • Stating that per occurrence will change/impact insurance charges and savings, so need adjustment; but without explanations • Not stating that the actuary's estimate of the increase in premium is too high.

QUESTION 15	
TOTAL POINT VALUE: 3.5	**LEARNING OBJECTIVE(S): B1, B6**
SAMPLE ANSWERS	
Part a: 2 points	

Sample 1

Slope = 1 / (α – 1) = .4467
Intercept = β / (α – 1) = 16,112
B = 36,069
α = 3.24

E[X] – E[X;20,000] = $(\frac{36,069}{3.24-1})$ (1 - $(\frac{36,069}{20,000+36069})^{3.24-1}$ = 10,108
E[X] = 16,102

Aggregate
E[A] = (15)(16,112) = 241,530
E[A_D] = (15)(10,108) = 151,620
E[A_e] = 241,530 - 151,620 = 89,910

B = 50,000 + 1.06(89,910) + (89,910)1.1=154,296

Sample 2

y = 0.4467*20000 + 16112 = 25,046
e(20,000) = (20K + β) / (α – 1) = 25,046
e(0) = 16112 = β / (α-1)
16112(α-1) = β
(20000 + 16112(α-1)) / (α-1) = 20000/(α-1) + 16112 = 25046
α = 3.239
β = 36.068.95
S(20K) = $(\frac{36068.95}{20000+36068.95})^{3.239}$ = 0.2396
Excess loss = 25046*15*S(20K) = 25046 * 3.594 = 90,015
Basic Premium = 50,000 + 0.06*90015 + 1.1*90015 = 154,417.77

Part b: 1 point

Sample 1

Expected severity below limit = $(\frac{36068.95}{3.239-1})(1-(\frac{36068.95}{20000+36068.95})^{3.239-1})$ = 10,109.95
Expected losses below limit = 10,109.95 * 15 = 151,649
Actual losses below limit = 151,649 / 2 = 75,825

Solve for Retro premium:
= (basic premium + LCF*actual loss below limit)/(1-commission-tax)
= (154,417.77 + 1.1*75,825)/(1-0.09-0.08)
= 286,536

Part c: 0.5 point

Sample 1

Yes, R^2 is high and the fitted line is close to the observed values, indicating that the line fits well. However, the slope of the regression line is sensitive to the size of the largest claims, and the calculated distribution parameters could be significantly affected by changes in just a few of these numbers.

Sample 2

The regression line is linear and upward sloping which matches the shape of a typical Pareto excess severity distribution. Further, the fitted line is reasonably close to the actual data and the quality of the fit is also confirmed by a high R^2 value.

EXAMINER'S REPORT

Candidates were expected to demonstrate understanding of an excess severity function and the components of a retrospective rated policy, including the relationship between them.

Part a

Candidates were expected to translate the excess severity function given into the alpha and beta parameters of a pareto distribution. Using that information, candidates were then expected to calculate the expected excess losses, and to use the expense assumption given to determine the basic premium.

If candidates were unable to solve for alpha and beta and instead made an assumption of their values, partial credit was given if subsequent calculations were correct.

Some candidates did not include excess losses in the basic premium calculation. However, if these candidates later included a separate excess loss premium in their calculation of the retro premium in part b, then full credit was given for part a.

Common mistakes included:

- Not solving for alpha or beta
- Mistaking the fixed overhead expenses of $50,000 for e, the expenses underlying a guaranteed cost premium
- Not multiplying the expected excess severity by the expected frequency in the excess loss calculation
- Forgetting to adjust by the profit load
- Not applying the LCF to expected excess loss
- Using expected limited loss instead of excess loss in the basic premium calculation
- Not including excess losses in either the basic premium or retrospective premium.

Part b

Candidates were expected to use the alpha and beta parameters calculated in part a to calculate the expected losses below the limit (or use the value of limited expected losses from part a if already calculated). Candidates were then expected to use the information provided in the problem to calculate actual losses from expected losses below the limit. Candidates who used an incorrect value for expected losses below limit (calculated in part a) were given credit in part b if they correctly calculated actual losses by dividing this value by 2.

Candidates were then expected to calculate the final retrospective premium by using the actual limited losses in the retro premium formula, which includes the basic premium, converted actual losses, and the tax multiplier.

Commons mistakes included:

- Using unlimited expected losses above the limit to calculate actual losses
- Not including frequency in the actual loss calculation
- Not applying the LCF to actual limited loss
- Incorrectly calculating the tax multiplier.

Part c

Candidates were expected to comment on the actuary's decision to fit a regression line to excess severity to estimate losses using the shifted Pareto distribution. This included commenting on the fit of the line vs. the plotted points, the R^2 value included in the question, and/or the shape of the fitted line.

Candidates were also expected to discuss the sensitivity of the slope of the regression line to the size of the largest claims, and how the distribution parameters could be impacted by a change in any of these larger losses. Candidates needed to make appropriate comments that discussed the overall quality of either the choice of distribution, the fit of the distribution, and/or the sensitivity of the fit to the size of the largest claims.

Common mistakes included:

- Incorrectly stating the shape of the fitted line did not match the shape of the Pareto distribution when fitting a regression line to excess losses
- Stating the R^2 value was low and not indicative of a good fit
- Not offering enough detail to assess the actuary's decision.

QUESTION 16	
TOTAL POINT VALUE: 3.0	**LEARNING OBJECTIVE(S): B5, B6**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

$r = \frac{A_D}{E(A_D)} \qquad E(A_D) = \frac{600{,}000}{5} = 120{,}000 \quad \phi^{LM}(r_{max}) = 0$

$\phi^{LM}(r_i) = \phi^{LM}(r_{i+1}) + (r_{i+1} - r_i)(\%\, risk\ above)$

Risk	r_i
1	0.25
2	0.5
3	0.75
4	0.5
5	3

r	# risks	# risks above	% risks above	$\phi(r)$
0	0	5	100%	1
0.25	1	4	80%	0.75
0.5	2	2	40%	0.55
0.75	1	1	20%	0.45
1	0	1	20%	0.4
1.5	0	1	20%	0.3
2	0	1	20%	0.2
2.5	0	1	20%	0.1
3	1	0	0%	0

$\phi(3) = 0$

$\phi(2.5) = \phi(3) + (3 - 2.5) \times 0.2$

Sample 2

$$E(A_D) = \frac{600k}{5} = 120k$$

r	$r \cdot E[A_D]$	# risk above	% risk above	$\phi(r)$
0	0	5	100%	1
0.5	60k	2	40%	$\frac{(90-60)+(360-60)}{(5)(120)} = 0.55$
1	120k	1	20%	0.4
1.5	180k	1	20%	0.3
2	240k	1	20%	0.2
2.5	300k	1	20%	0.1
3	360k	0	0%	0

Part b: 1.5 points

Sample 1

- 1 – adv: Stable premium, known at policy inception. Disad: no reduction in premium in case of good loss experience
- 2 – adv: minimize cost as compared to GCP since insured is responsible for losses below deductible, so premium smaller → less taxes paid. Disad: cash flow less stable than GCP since it depends on the number of claims and severity
- 3 – adv: cover all losses as compared to LDD where there is a deductible on each claim. Disad: cash flow less predictable as premium is adjusted as losses develop

Sample 2

- GCP – advantage: the GCP will have the most stable, predictable cash flows. Disadvantage: the cost up front will be higher than the other two policies
- LDD – advantage: retain lower losses while having protection against larger losses. Disadvantage: potential to pay a lot if there are a lot of claims below deductible
- Retro policy – advantage: the reduced severity from their program will likely result in lower actual losses driving down premium. Disadvantage: potential to pay a lot if loss experience is bad

Sample 3

	Advantage	Disadvantage
GCP	Cost is known up front (not subject to change)	Doesn't help to minimize costs or recognize good loss experience
LDD	Low upfront cost	Costs are uncertain as compared to a GCP (losses unknown to be paid)
Retro	Incentive for safety (better safety = lower loss = lower cost)	Premium is subject to fluctuations (not stable throughout the policy period)

EXAMINER'S REPORT

Candidates were expected to be able to calculate a Limited Table M, and then discuss the benefits and drawbacks of guaranteed cost plans, large dollar deductible plans, and retrospective rating plans from the perspective of an insured based on their goals and situation.

Part a

Candidates were expected to construct a Limited Loss Table M using the loss data provided for the given entry ratio range.

Common mistakes included:

- Using unlimited losses or unlimited expected losses or both
- Calculating the excess ratio for the per-occurrence limit, and using this as the insurance charge for r = 3
- Building the table using only the required 7 entry ratios, and not accounting for the percentage of risks above changing at 0.25 and 0.75 as well

- Failing to include a demonstration calculation or formula for how the charges were determined.

Part b

Candidates were expected to provide an advantage and a disadvantage for each of the three policies based on the priorities of the insured.

Common mistakes included:

- Stating that the guaranteed cost policy would be the least expensive, or would have unstable cash flows
- Stating the LDD policy minimizes cost without making it clear that "cost" referred to premium or justifying why the overall cost was lower (such as the deductible losses decreasing due to the severity reduction program)
- Stating general features of a policy without indicating why that feature was either an advantage or disadvantage (such as the need to pay losses under a deductible, or that claim adjustment is handled by the insurer)
- Having disadvantages for the LDD or Retro stating that less or no loss would fall into the excess layer or over the aggregate limit, without tying it to the pricing of these layers
- Misunderstanding the nature of the LDD per-occurrence limit or the Retro aggregate limit, and the loss layers that impacted the insured vs. the insurer.

QUESTION 17	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE(S): C4**
SAMPLE ANSWERS	
3 points	

Sample 1

Year 1

	Tech Ratio	P	
0	43 + 35 = 78	0.15	Yr 1 expected tech ratio =90.075%
50	57 + 35 – 5 × 0.7 = 88.5	0.25	
60	68 + 27.5 – 3 = 92.5	0.45	
80	85 + 12.5 = 97.5	0.15	

Year 2 (15% of the time)

	Tech	
0	43 + 35 = 78	$\text{Yr } 2_{15\%} = 87.2\%$
50	57 + 35 – 0.5 × 12 = 86	
60	68 + 27.5 – 8 = 87.5	
80	85 + 12.5 = 97.5	

Yr 2 Total = 0.85 × 90.075 + 0.15 × 87.2 = 89.6%

Sample 2

$ELR_{YR1} = ELR_{YR2}$ = 43% (0.15) + 57% (0.25) + 68% (0.45) + 85% (0.15) = 64.05%

Year 1 Expected Commission	
Range of LR	Commission @ Avg LR
0 – 50	35%
50 – 60	(65% - 57%) (0.5) + 27.5% = 31.5%
60 – 80	(80% - 68%) + 12.5% = 24.5%
80 +	12.5%

Expected Yr1 Commission = 0.15 (35%) + 0.25 (31.5%) + 0.45 (24.5%) + 0.15 (12.5%) = 26.025%

Year 2 Expected Commission Only the 80+ group will be affected (we will add 5% to the LR's for the purpose of calculating commission)		
Range of LR	Modified Avg LR	Commission
0 – 50	43% + 5% = 48%	35%
50 – 60	62%	29%
60 – 80	73%	19.5%

80 +	86%	12.5%

Expected commission for 80+ group = 0.15 (35%) + 0.25 (29%) + 0.45 (19.5%) + 0.15 (12.5%) = 23.15%
Expected Yr2 Commission = 0.15 (23.15%) + (1 – 0.15) (26.025%) = 25.59%
Expected Technical Yr1 = 26.025% + 64.05% = 90.075%
Expected Technical Yr2 = 89.64%

Sample 3

Range	Prob	Avg LR	Commission	Yr1 Tech Ratio = 0.15(0.43 + 0.35) +0.25(0.57 + 0.315) + 0.45(0.68 + 0.245) + 0.15(0.85+0.125) = 0.90075
0 – 50	0.15	43%	35%	
50 – 60	0.25	57%	31.5%	
60 – 80	0.45	68%	24.5%	
80 +	0.15	85%	12.5%	

Next calculated expected carryforward. Assume only carryforward LR above max.
E(carry forward) = 0.15(0.85 – 0.80) = 0.0075

Year 2

Range	Prob	Avg LR	LR for Comm	Commission	Technical Ratio
0 – 50	0.15	43%	43.75%	35%	78%
50 – 60	0.25	57%	57.75%	31.125%	88.125%
60 – 80	0.45	68%	68.75%	23.75%	91.75%
80 +	0.15	85%	85.75%	12.5%	97.5%

Expected Year 2 Technical Ratio = 0.15(0.78) + 0.25(0.88125) + 0.45(0.9175) + 0.15(0.975) = 89.64%

EXAMINER'S REPORT

Candidates were expected to determine the effect of a carryforward provision on the cost of a reinsurance contract. Candidates needed to determine the technical ratio for both year 1 and year 2, taking the possible carryforward provision into account.

To receive full credit, candidates were expected to calculate the technical ratios for both years. There were several mathematically equivalent ways to do this. Most candidates correctly calculated the technical ratio for year 1, but struggled with the calculation of the second year.

Common mistakes included:

- Ignoring Year 2 completely and only calculating the technical ratio for Year 1
- Assuming that there was no possibility of the carryforward provision being triggered in year 2, and that therefore the technical ratio in year 2 would equal that of year 1
- Calculating Year 1 and Year 2 correctly but forgetting to weight together for the final answer.

QUESTION 18	
TOTAL POINT VALUE: 5.5	**LEARNING OBJECTIVE(S): C**

SAMPLE ANSWERS

Part a: 0.5 point

Sample 1

(100 x 0.25 + 120 x 0.3 + 150 x 0.45 + 80 x 0.24 + 100 x 0.36) / 550 = .334

Sample 2

Use a simple average of 5 year data

Loss Cost = (0.25 + 0.3 + 0.45 + 0.24 + 0.36) / 5 = 0.32

Part b: 1 point

Sample 1

G(1m / 1m) – G(500k / 1m) = 1 – 0.6372 = 0.3628

Pure Risk Prem =(75 + 90 + 135 + 60 + 80) / 550 = 0.8

Loss Cost = 0.3628 x 0.8 = 29.024%

Sample 2

Since max possible loss is 1M, I assume X represents ground up loss net of the quota share. Otherwise, X > 1 at limit of excess treaty, which violates the condition that 0 <= X <= 1 when using max possible loss

G(500K / 1M) = G(0.5) = (1 – 0.32428^0.5) / (1 – 0.32428) = 0.6372

G(1) = 1

Avg Loss Ratio Net of QS = (3 x 0.75 + 0.8 + 0.9) / 5 = 0.79

Loss Cost = 0.79 (1 – 0.6372) = 0.287

Part c: 0.5 point

Sample 1

0.8 x 33.4m + 0.2 x 29.024m = 32.5248m

Sample 2

Loss Cost = 0.8 x 0.32 + 0.2 x 0.287 = 0.313

Part d: 2 points

Sample 1

E[Loss Ratio] = 80%

Ceded using cred wt = 32.52%

Retained = 80 – 32.52 = 47.48%

Ceded Prem = 32.5248m / 0.9 = 36,138,667

Profit = 100m – (80m – 32.5248m) – 36.138667m – 15m = 1,386,133

Sample 2

Use loss cost from part c to estimate ceded losses.

Expected Ceded Loss to XS = 100M x 0.313 = 31.3M

0.9 = 31.3M / XS Premium

XS Premium = 34.78M
Ceding Commission = (100 / 0.75 – 100) x 0.15 = 5M
Profit = 100M + 5M – 34.78M – 100M (0.79 – 0.313) – 0.15 (10M / 0.75) = 2,522,222

Part e: 1 point

Sample 1

Reinsurer's Profit = Max[(1 – Ceded LR – Ceding Commission – Margin), 0]
= Max[(1 – Ceded LR – 15% - 5%), 0]
Profit Commission = 100% * Reinsurer's Profit

AY	Reinsurer's Profit %	Profit Commission
2014	Max(1-75%-15%-5%, 0) = 5%	5% = 100% * 5%
2015	Max(1-75%-15%-5%, 0)= 5%	5%
2016	Max(1-90%-15%-5%, 0) = 0	0
2017	Max(1-75%-15%-5%, 0)= 5%	5%
2018	Max(1-80%-15%-5%, 0) = 0	0

Weighted Average = $\frac{(5\%*100 + 5\%*120 + 0 + 5\%*80 + 0)}{100 + 120 + 150 + 80 + 100}$ = 5% * 300/550 = 2.73%

Sample 2

Ceded Premium = $\frac{\text{On Level Subject EP (net of QS)}}{(1 – 25\%)}$ * 25%
Ceded Loss = Ceded Premium * ULR
Ceding Commission = 15% * Ceded Premium
Profit Margin = 5% * Ceded Premium
Profit Commission = Max(0, Ceded Premium – Ceded Loss – Ceding Commission – Profit Margin)

AY	On Level Subject EP	Ceded Premium	Ceded Loss	Ceding Commission	Profit Margin	Profit Commission
2014	100	33.33	25	5	1.67	1.67
2015	120	40	30	6	2	2
2016	150	50	45	7.5	2.5	0
2017	80	26.67	20	4	1.33	1.33
2018	100	33.33	26.67	5	1.67	0
Total		183.33				5

Weighted Average = 5 / 183.33 = 2.73%

Part f: 0.5 point

Sample Responses for Advantage of Proposed Option 1 (Profit Commission) vs. Option 2

- Proposed profit commission allows the insurer to benefit from favorable results with good underwriting performance

- The insurer can retain a significant amount of profit if the ceded business performs well
- Profit commission can provide incentive for risk control and get money back if that risk control is successful

Sample Responses for Disadvantage of Proposed Option 1 (Profit Commission) vs. Option 2

- Higher ceding commission is paid upfront so there is cash flow benefit for the insurer
- Expected commission under option 1 based on part e is 2.73% + 15% = 17.73%, which is lower than the 20% guaranteed commission under option 2
- As shown in part e, the expected profit commission is 2.7%, which is lower than the added 5% ceding commission under option 2, therefore on average option 2 is better
- Profit commission is not as stable as higher ceding commission which is guaranteed.

EXAMINER'S REPORT

Part a

Candidates were expected to take the historical layer loss ratios with corresponding premiums to derive the weighted average expected loss cost. Full credit was given to both weighted and straight averages.

Common mistakes included:

- Failing to recognize the given table is net of Quota Share
- Applying ultimate loss ratio on top of layer loss ratio
- Not taking the average for expected figures

Part b

Candidates were expected to apply the given exposure curve to derive the expected loss percentage in the layer and then apply it to the subject loss ratio to determine the exposure loss cost.

Common mistakes included:

- Applying Excess Layer Ceded Loss Ratio as Subject Loss Ratio
- Forgetting to apply Subject Loss Ratio on loss in layer
- Applying ceding ratio on maximum possible loss.

Part c

Candidates were expected to apply the correct credibility weights to the experience loss cost from part a and the exposure loss cost from part b.

A common mistake included:

- Applying the wrong credibility.

Part d

Candidates were expected to calculate premium, losses and expenses after applying the terms of two treaties and then combine them in the expected underwriting profit provision. Candidates received full credit for deriving gross and ceded components and then combining them into net figures. Candidates could also attempt to calculate the u/w provision in a percentage form as long as all components were converted to a consistent base.

Common mistakes included:

- Missing components of premiums, losses or expenses (e.g. including the benefit of quota share but not excess of loss treaty)
- Mixing up gross and net figures or applying factors to incorrect base (e.g. given subject premium net of quota share as gross premium or applying PPR loss cost to subject losses instead of subject premiums)
- Using inconsistent calculations for different components of the same treaty (e.g. calculating ceded PPR losses based upon historical loss cost from part a while applying loss cost from part b or c to calculate premium ceded to the same treaty)

Part e

Candidates were expected to calculate the 5 year weighted average profit commission as a ratio to ceded premium with the proposed structure from Option 1 using the historical experience data provided.

There were two ways to calculate the weighted average profit commission ratio that received full credits. Candidates could:

- Calculate the profit commission percentages for each accident year and get weighted average ratio using either gross premiums or ceded premiums as weights since both provide the same weights under a QS treaty
- Calculate the profit commission dollars for each accident year based on ceded premiums, sum them up across all five years and divide the total profit commissions by the total ceded premium. Candidates who consistently used gross premiums instead of ceded premiums also received full credit as it made no difference to the final ratio under a QS treaty.

Common mistakes included:

- Forgetting to include the 15% ceding commission when calculating profit commission
- When calculating the profit margin, incorrectly multiplying the 5% margin with either the reinsurer's profit or ceded premium net of ceding commissions, instead of multiplying it with the ceded premium
- Calculating profit commission using 5-year total ceded premium, losses, and commissions, instead of calculating it for each AY individually which would have captured any negative profit commissions and applied a floor of zero to them
- Calculating a simple 5-year average of the profit commission instead of a weighted average.

Part f

Candidates were expected to understand different commission structures between the primary insurer and reinsurer as well as the purpose and mechanics of the profit commission structure as described in Clark.

Candidates could talk from either Option 1 or Option 2's standpoint and received full credits as long as both one advantage (over the other option) and one disadvantage (over the other option) were correctly provided.

Common mistakes included:

- Simply stating that the insurer has potential to receive higher commission under Option 1 without explanation
- Stating that the insurer is giving up 20% guaranteed ceding commission for the profit commission, not recognizing the insurer is only giving up 5% additional ceding commission over the existing 15% ceding commission
- Stating that insurer might get no commission under option 1, without considering the fixed 15% ceding commission
- Assuming that profit commissions are paid by the insurer to the reinsurer
- Stating something too vague or generic that isn't necessarily an advantage/disadvantage over the other option
- Talking from the reinsurer's perspective with something that cannot be translated into a proper advantage/disadvantage from the insurer's perspective

QUESTION 19	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): C**
SAMPLE ANSWERS	
Part a: 0.5 point	
Sample 1 By the graph, >= 750M loss has return period >= 25 years 1/25 = 4% *Sample 2* 1/25 = 4% *Sample 3* Pr[Ret > 750M] on a Gross basis ⇨ Return period = 1-in-25 years ⇨ PR[Ret>750M] = 4%	
Part b: 0.75 point	
Sample 1 Option 1 is the Quota Share treaty In a Quota Share treaty, losses are shared in the same proportion across all risks, there fore CV(Gross) = CV(Retained) = CV(Ceded) QS Percentage is 10% *Sample 2* Option 1 is the Quota Share treaty – it has the same CV on gross and retained loss. At 1-in-25 year, QS % = 1 – 675/750 = 10% *Sample 3* The quota share option is option 1 The shape of retained loss by return period is identical just reduced by 10% so the QS% is 10%	
Part c: 0.5 point	
Sample 1 • The coefficient of variation is much higher for the ceded portion of losses for option 3 (CV = 169%) compared to that of Option 1 (CV = 25%) • This means that there is a lot more uncertainty in the amounts that the reinsurance treaty will cover. This requires a higher risk load for the reinsurance premiums *Sample 2* • Option 3 picks up the volatile part in the tail, compare to Quota Share option 1 • Reinsurer needs more capital to support losses that are riskier/more volatile. The profit margin will be higher. *Sample 3*	

- Option 3 may have a higher risk load since there could be more volatility in the ceded loss, especially since CoV in ceded loss is 169%

EXAMINER'S REPORT

Candidates were expected to demonstrate understanding of various reinsurance options by using the graph and data table provided. Candidates were expected to read a return period off the graph, understand how to use the information to both pick which option was a quota share and calculate the quota share percentage, and then analyze which option was likely to be costliest.

Part a

Candidates were expected to know how to read return period from the graph and calculate the probability that the insurance company retains losses above a certain point.

Common mistakes included:
- Not identifying a return period from the graph
- Incorrectly calculating the probability from the return period.

Part b

Candidates were expected to identify the quota share treaty from the graph and calculate the quota share percentage.

Most candidates were able to identify which option was the quota share treaty, but many of them did not state the quota share percentage. If candidates listed data points from the curve and used that to show the AAL or ceded % are identical for option 1, credit was given.

Common mistakes included:
- Not providing the quota share percentage
- Stating that Option 2 is the quota share because the sum of ceded and retained CV equals 1
- Stating that Option 3 is the quota share because the shape of retained loss is the same as gross loss.

Part c

Candidates were expected to justify why the reinsurance premium may be higher for one treaty compared to another.

Common mistakes include:
- Not providing justification on why volatility/tail risk coverage would lead to higher reinsurance premiums
- Claiming that Option 3 is more expensive because it has higher AAL.